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Stiffness method for pipeline analysis

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ABSTRACT

Analysis of structures is done in two ways reliance matrix and stiffness matrix methods. In the soft method, the unknowns of the problem are forces. In the soft methods, we need an additional equation for the number of statically indeterminate degrees. In our hardness method, the unknowns are the deformation of the structure, and after obtaining them, the unknown forces of the structure can be obtained. Also, for structures that have a low degree of uncertainty but a high degree of freedom, the soft method is easier, but for structures that have a low degree of freedom, the hard method is good. In this article, we have investigated the structure related to the pipelines of a station, such as the gas pressure boosting station, using the stiffness method. Pipeline systems under thermal loads are frequently used in many systems are relate to in oil and gas like power stations or pumping houses and petrochemicals and gas stations. Unlike the analytical center of elastic method, which can analyze only a single branch of pipeline with members the coordinate system installed parallel to the pipe, the stiffness method removes these kinds of limitations. In this article, the stiffness method is introduced to analyze pipeline systems. Based on this method, the piping system may include any numbers of pipe branches. The straight elements of pipe in the branch may have any general orientation, and the bend elements may have any arbitrary angle. The computer program designed based on this method may compute the nodal displacements at the ends of the straight or bent elements and present three components for loads and three for moments. Once the free-body diagram of each piping element with nodal forces and moments is drawn, the engineering codes are employed to design the piping system for safe operation. By using this method every kind of pipelines including constraint and unconstraint could be analysed. As we know in powerplant and pipeline systems, thermal stresses are one of the major problems. Every year this phenomenan cause many damages for industries then computing and predicting the effect of thermal loads are very essential for engineeres. Stiffness method is a general and simple method for computing nodal forces for piping or pipeline systems and every kind of pipelines with any geometric plan and every number of branch of pipes. Maximum forces and moment could be computed in every point of system and then the best position for installation of hangers and foundations can be determined.

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1. Introduction

Piping systems, like those built in refineries, transfer fluids or gases at high-pressure and high temperatures, and any failure or fracture in these systems may cause very huge damage. Advanced codes in design have been created to make safety instructions for designers. Piping systems at first installed in a reference temperature. In system operation conditions, by temperatures rising, forces and bending moments are created at the ending of connecting expansion and contraction. Due to the imposed constraintsthe effect thermal expansions can create stresses, that, not only doese not take care, but also cause damage.

The resulting forces and moments created in the system should be calculated to design the piping system. Following the engineering codes, stress analysis is performed for safe working conditions. The design engineering codes need for calculating of the forces and moments of the system created by the imposed boundary conditions. Once the forces and moments are obtained, the free-body diagram of each member of the piping system, straight and bend elements, are drawn with forces and moments, and the engineering code is used to do the stress analysis.

There are different calculation methods, both moments and forces produce in a piping system. As an analytical method, one may refer to the method of elastic center. The stiffness and FEM are one of the essential and general techniques for obtaining the end forces and moments. While the elastic center method is confined to only a single pipe of the of the piping, and the coordinate systems must construct parallel to the piping system, but we can eliminate every limitations by using the method of stiffness. The method of stiffness any number of pipes in the piping system and with any orientation of the pipes mounted in the system can easily control and calculate their forces exerted to them at ends. A method for easy and manual calculations of piping system forces and moments is given by Shinger [1]. The methods for more complicated analysis need computer programming, as given in [2]. The pipeline analyses focused on the marine pipeline systems may be found in [3].

In a pressure boosting station, the gas output from the compressor comes out with high pressure and temperature and enters the piping system inside the station. In some places, this piping system is on the support and in some other places it is bound by different systems. According to the type of turbo compressor used in each station, as well as the gas transfer capacity of the station, operating conditions in different seasons of the year, various problems can arise for the piping systems of each station, for example, the increase in the temperature of the exhaust gas can affect the pipe lining. The output of the station has an effect and causes the cover to burn, which itself causes damage to the cathodic protection system. An increase in pressure can cause the vibration of the compressor output and cause fatigue and ultimately the failure of the output pipe from the compressor. The surge in the turbine itself causes severe vibrations and as a result damage to the piping system. The poor performance of the control valves inside each station is another factor for the appearance of various defects in the piping system. Therefore, the creation of various defects in the systems of piping for each station is related to many factors. In the analysis of many defects created in piping systems inside a station, including defects that lead to failure, the use of finite element analysis is very useful. With this method, we can identify the high stress points and as a result we can identify displacements and other parameters that can cause destruction or explosion in this network. Apart from using the finite element method to investigate the result of different moments and loads applied to the system of piping, there are other methods, such as the methods of using the softness and hardness matrix. As it was said at the beginning, if we use the softness matrix method, we can directly analyze and evaluate the incoming forces as well as the created moments, and by receiving the results, we can identify the points where the most forces and moments are applied and then control them. but another method that can be used is the use of the stiffness matrix method. As stated in this problem, due to the application of different loads and forces, as well as the presence of high temperature of the gas exiting the turbine, we face a multi-variable problem. By using this method, the moments and forces generated in different points, especially the joint points, as well as elbows and pipes that are under bending moment, are obtained, and after analyzing and checking the amount of force, these points can be strengthened.

If the main aim of structural analysis is to specify the displacement of the two ends of the element, or in other words, to specify the nodes displacements of the structure, in this case, analysis of the structural is done by method of displacement or method of stiffness . In the method of stiffness, the unknowns include changing the nodes locations and The number of obtained equations is equal to the degree of freedom (DOF) of all the nodes of the structure.

Therefore, in the method of stiffness, first, the change in the locations of specific points is determined, especially in the nodes of the structureand and the internal forces have been calculated in the next step.

In this method, equations are created between the forces and the changes in the locations of the structure at the two levels of the element and the whole structure. So, in a summary, the difficulty method includes the following general steps. 1- Determining a set of displacements of the structural system 2- Writing force-displacement relationships 3- Satisfying compatibility conditions 4- Solving the equations and obtaining the displacement of the structural system 5- Obtaining member forces and support reactions.

In the matrix analysis of the structures in a difficult way, in fact, the mentioned equations are extracted in the matrix form and the basics of matrix algebra are used. These equations are matrix type and include the forces in the vector form and the displacement vector, and at the same time, there will be another matrix known as the stiffness matrix, which depends on the geometry of the structure, the geometric properties of the members' materials, the type of connections in the structure, the supports, the way the members are connected, etc. . has it.

This paper presented the stiffness method based on the structural matrix analysis. The method of stiffness is basically concluded from the method of structural analysis under different mechanical loads. On the other hand the thermal loads act as like as the mechanical loads, and we can use them in the method of stiffness for structure analysis. The first part of the paper presents the essential governing equations and relations of the straight and bent pipe members. In the second part, the flow chart of a program that uses the stiffness method to calculate the end forces and moments is given. An example is considered and solved with stiffness the method, and the results have been compared with the results which find of Ansys software.

2. Stiffness Method for Piping Systems

Consider a deformable body under the action of forces P_i ;i=1,2,..,n, as shown in Fig. 1. Based on the Castiglioni theorem, once the energy of deformation is calculated by Eq. (1), the deformation under the applied force is calculated as

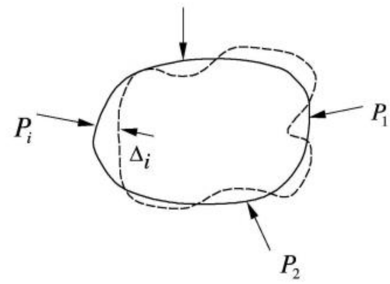

Fig. 1. An elastic body under the action of forces

$$U = \frac{1}{2}\left[P_1\Delta_1 + P_2\Delta_2 + \ldots + P_n\Delta_n\right] \quad (1)$$

$$P_i = \frac{\partial U}{\partial \Delta_i} = \frac{\partial P_1}{\partial \Delta_i}\Delta_1 + \frac{\partial P_2}{\partial \Delta_i}\Delta_2 + \ldots + \frac{\partial P_n}{\partial \Delta_i}\Delta_n \quad (2)$$

$$\begin{bmatrix} P_1 \\ \text{M} \\ \text{M} \\ \text{M} \\ P_n \end{bmatrix} = \begin{bmatrix} \frac{\partial P_1}{\partial \Delta_1} & \text{L} & \frac{\partial P_n}{\partial \Delta_1} \\ \text{M} & \text{O} & \text{M} \\ \frac{\partial P_1}{\partial \Delta_n} & \text{L} & \frac{\partial P_n}{\partial \Delta_n} \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \text{M} \\ \text{M} \\ \text{M} \\ \Delta_n \end{bmatrix} \quad (3)$$

Equation (3) we can write it in matrix form as $P=KX$, where K is the stiffness matrix and is symmetric. For a branch of pipeline between points i and j (Fig. 2), we can write Eq. (3) as below:

$$P_i = K_{ii}\Delta_i + K_{ij}\Delta_j \quad (4)$$

$$P_j = K_{ji}\Delta_i + K_{jj}\Delta_j \quad (5)$$

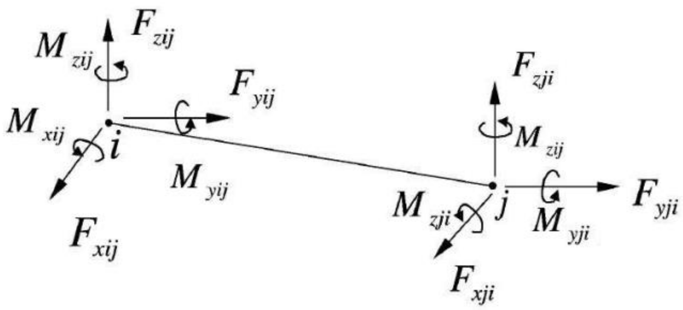

Fig. 2. A Branch between point i and j

Which Eq. (4) is for point *i* and Eq. (5) is for point *j* and P, Δ is assumed as:

$$P_i = \langle F_{xi} \quad F_{yi} \quad F_{zi} \quad M_{xi} \quad M_{yi} \quad M_{zi} \rangle \tag{6}$$

$$\Delta_i = \langle \Delta_{xi} \quad \Delta_{yi} \quad \Delta_{zi} \quad \theta_{xi} \quad \theta_{yi} \quad \theta_{zi} \rangle \tag{7}$$

if we consider $\Delta_j = 0$ then we have Eq. (8), and by inverse K_{ii} in Eq. (8) then Eq. (9) is concluded. Which D_{ii} is the flexibility matrix of point *i*.

$$P_i = K_{ii}\Delta_i \tag{8}$$

$$\Delta_i = K_{ii}^{-1} * P_i = D_{ii} * P_i \tag{9}$$

To transform a vector from a coordinate system (XYZ) to another (X'Y'Z'), use transformation and rotation matrix in FEM. It's necessary to transform forces and displacements in local coordinates to global coordinates and consider their effect trulyto transform them in local coordinates to global coordinates and truly consider their effect. We assume R and H as rotation matrix and transfer matrix respectively and H for transfer forces from point *i* to point *j* is:

$$H_{ij} = \begin{bmatrix} I & O \\ \begin{matrix} 0 & -(Z_i - Z_j) & (Y_i - Y_j) \\ (Z_i - Z_j) & 0 & -(X_i - X_j) \\ -(Y_i - Y_j) & (X_i - X_j) & 0 \end{matrix} & I \end{bmatrix} \tag{10}$$

and can be shown for each R and H:

$$R^{-1} = R^{*} \tag{11}$$

$$H_{ij} = H_{ji}^{-1} \tag{12}$$

Stiffness Matrix of a straight element is calculated from Castiglioni theorem:

$$D_{ii} = \begin{bmatrix} \frac{l}{EA} & 0 & 0 & 0 & 0 & 0 \\ & \frac{l^3}{EA} & 0 & 0 & 0 & \frac{l^2}{2EA} \\ & & \frac{l^3}{EA} & 0 & \frac{-l^2}{2EA} & 0 \\ & & & \frac{(1+v)l}{EA} & 0 & 0 \\ & Sym & & & \frac{l}{EA} & 0 \\ & & & & & \frac{l}{EA} \end{bmatrix} \tag{13}$$

and Stiffness Matrix of a curved member is calculated too:

$$D_{ii} = \begin{bmatrix} d_{11} & d_{12} & 0 & 0 & 0 & d_{16} \\ & d_{22} & 0 & 0 & 0 & d_{26} \\ & & d_{33} & d_{34} & d_{35} & 0 \\ & & & d_{44} & d_{45} & 0 \\ & Sym & & & d_{55} & 0 \\ & & & & & d_{66} \end{bmatrix} \tag{14}$$

$K = 1 + \dfrac{9}{12\,h^2 + 1}$ t : pipe thickness

$h = \dfrac{tR}{r_m^2}$ r^m : the average of pipe radius

$d_{11} = \dfrac{KR^3}{EI}\left(\dfrac{3\varphi}{2} - 2\sin\varphi + \dfrac{\sin 2\varphi}{4}\right)$ R : the curve radius

$$d_{12} = \frac{KR^3}{EI}\left(\frac{3}{4} - \cos\varphi + \frac{\cos 2\varphi}{4}\right)$$

$$d_{33} = \frac{R^3}{EI}\left[(3+3v+K)\frac{\varphi}{2} - 2(1+v)\sin\varphi + (1+v-K)\frac{\sin 2\varphi}{4}\right]$$

$$d_{34} = \frac{R^2}{EI}\left[-(1+v+K)\frac{\varphi}{2} + (1+v)\sin\varphi + (K-1-v)\frac{\sin 2\varphi}{4}\right]$$

$$d_{35} = \frac{R^2}{EI}\left[-(3+3v+K)\frac{1}{4} + (1+v)\cos\varphi + (K-1-v)\frac{\cos 2\varphi}{4}\right]$$

$$d_{44} = \frac{R}{EI}\left[(1+v+K)\frac{\varphi}{2} + (1+v-K)\frac{\sin 2\varphi}{4}\right]$$

$$d_{45} = \frac{R}{4EI}(K-1-v)(1-\cos 2\varphi)$$

$$d_{55}=\frac{R}{EI}\left[(1+v+K)\frac{\varphi}{2}+(K-1-v)\frac{\sin 2\varphi}{4}\right]$$

$$d_{66}=\frac{KR\varphi}{EI}$$

To assemble the stiffness matrix of a branch, the superposition method is used. As is shown in Fig. 3 displacement in point i is the summation of cases n on Fig. 3. for calculating the displacement of Fig. 3-e:

$$P_2 = H_{i2}P_i \tag{15}$$

$$\Delta_{i2} = H_{i2}^{*}D_{22}\ H_{i2}P_i \tag{16}$$

Equation (15) calculates the effect of the transformation of forces from point i to point 2, and Eq. (16) calculates the displacement of point i when all forces apply in point 2. It can be written for all part of Fig. 3:

$$\Delta_{i3} = H_{i3}^{*}D_{33}\ H_{i3}P_i \tag{17}$$

$$\Delta_{i4} = H_{i4}^{*}D_{44}\ H_{i4}P_i \tag{18}$$

$$\Delta_{i5} = H_{i5}^{*}D_{55}\ H_{i5}P_i \tag{19}$$

so, displacement of point i is summation of Eq. (16) to Eq. (19). thereupon:

$$\Delta_i = \sum_{K=2}^{5}\Delta_{ik} = (\sum_{K=2}^{5}H_{iK}^{*}(D_{KK})H_{ik})P_i \tag{20}$$

and stiffness matrix of the branch in point i is:

$$D_{ii} = \sum_{K=2}^{5}H_{iK}^{*}(D_{KK})H_{ik} \tag{21}$$

The stiffness matrix for a branch of piping in point i is:

$$K_{ii} = D_{ii}^{-1} \tag{22}$$

Because this part of the piping is at static equilibrium, Boundary condition for node j is as follow:

$$P_j + H_{ij}P_i = 0 \Rightarrow P_j = -H_{ij}P_i \tag{23}$$

Substituting for P_j from the Eq. (4) gives:

$$P_j = -H_{ij}K_{ii}\Delta_i - H_{ij}K_{ij}\Delta_j \tag{24}$$

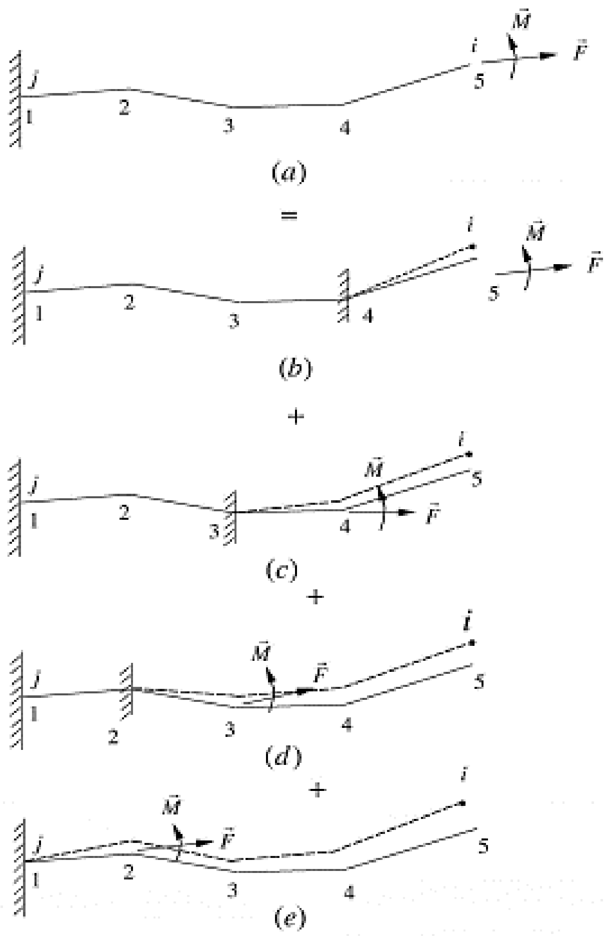

Fig. 3. assemble the stiffness matrix of a branch

Comparing Eq. (24) and Eq. (5) yields:

$$K_{ji} = -H_{ij} K_{ii} \quad (25)$$

$$K_{jj} = -H_{ij} K_{ij} \quad (26)$$

Since $K_{ij} = K_{ji}^{*}$ and for K_{ii}in Eq. (25) can be written $K_{ij} = -K_{ii} H_{ij}^{*}$ so at the end for K_{jj}we have:

$$K_{jj} = H_{ij} K_{ii} H_{ij}^{*} \quad (27)$$

3. Validation of Method

A Fortran program is developed to calculate and analyze stresses, strains, displacements and forces. All algorithm steps are implemented in this program, and the results are calculated. To verify the method and developed program, Ansys workbench 18.2 is used. This way, a pipeline branch with straight and bent parts was created in Ansys and specific anchor and temperate changes were applied.

It is assumed that the outlet radius of all pipes are 18.5 cm and their thickness are 0.68 cm. Elastic modules are 1680000 kg/cm^2 for all pipes. Calculated results by Ansys are presented at Table 1.

The problem also introduced to the Fortran program results are calculated. The calculated results by Fortran program are presented at Table 2.

Table 1 and Table 2 shows calculated forces and moments by Ansys and Fortran program, respectively, for each support, shown at Fig. 4. To estimate the error of results calculated by stiffness method, Eq. (28) is deployed, in which, S_A is Ansys output and S_F is Fortran result or output. The Error percentage (%e) is calculated for each output and presented at Table 3.

$$\%e(S) = \frac{|S_A - S_F|}{S_A} \times 100 \quad (28)$$

Table 1. Output data from Ansys

Force and Moment in Nodes	F_x (Kg)	F_y (Kg)	F_z (Kg)	M_x $(Kg.cm)$	M_y $(Kg.cm)$	M_z $(Kg.cm)$
1	-801.76	-788.63	-290.36	-64278.28	15675.84	151753.31
3	801.76	788.63	290.36	111202.85	-92419.97	-72876.65

Table 2. Output data from Fortran Program

Force and Moment in Nodes	F_x (Kg)	F_y (Kg)	F_z (Kg)	M_x $(Kg.cm)$	M_y $(Kg.cm)$	M_z $(Kg.cm)$
1	-786.9	-767.17	-288.20	-63939.23	16441.73	151743.2
3	786.9	767.17	288.20	108222.88	-91679.81	-72417.75

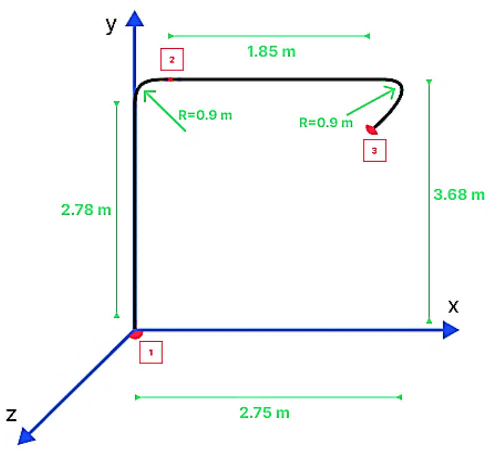

Fig. 4. A 3D branch of pipeline

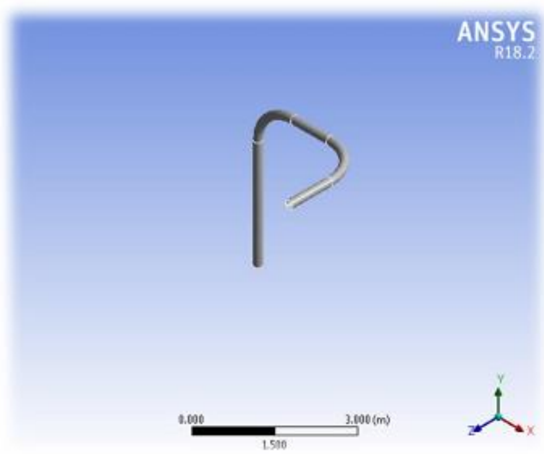

Fig. 5. Branch of pipeline in Ansys workbench

Table 3. % Error for each output

Node	$\%e\ (F_x)$	$\%e\ (F_y)$	$\%e\ (F_z)$	$\%e\ (M_x)$	$\%e\ (M_y)$	$\%e\ (M_z)$
1	1.85	2.72	0.74	0.53	4.89	0.01
3	1.85	2.72	0.74	2.68	0.80	0.63

4. Conclusion

In conclusion, this paper presented the stiffness method as a general technique for analyzing pipeline systems under thermal loads. The method allows for ofanalyzing any number of piping branches, with straight pipe elements in any orientation and bend elementsat any arbitrary angle. The step-by-step algorithm for deploying the stiffness method for a single branch is described in detail, and the local and global stiffness matrices are generated accordingly. A computer program was also computer program was also developed based on this method to calculate and analyze stresses, strains, displacements, and forces in pipeline systems. The program was then verified using Ansys Workbench 18.2, and the results were found to be accurate, with an error of less than 5%. Overall, the stiffness method and the associated program provide an efficient and effective way to design and analyze pipeline systems under thermal loads, ensuring safe and reliable operation.

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