

# Transmission loss allocation in bilateral or multilateral transaction-based markets

## Authors

Rahmat Aazami<sup>a</sup>  
Amin Moradkhani<sup>a\*</sup>

<sup>a</sup> Faculty of Engineering, Ilam University, Ilam, Iran

## ABSTRACT

*In this paper, the problem of transmission loss allocation has been studied and a new method for loss allocation in transaction-based markets has been proposed. To further this end, first transmission line loss equations were used with respect to bus injected currents. The share of each bus from the mentioned transmission line losses was determined. Then, this method was applied to the total network transmission lines. While considering the available transactions, the share of each bus from the total losses was acquired. The proposed method is based on the main network relations and no simplifying assumption has been used. Finally, the proposed method is based on a typical network.*

## Article history:

Received : 23 November 2016

Accepted : 16 June 2017

**Keywords:** Loss Allocation, Sending and Receiving Power, Multilateral Transaction.

## 1. Introduction

In power systems, some percentage of the transmitted power is always wasted due to various factors. The main cause of these losses is a result of the flow of current in ohmic resistance in the transmission lines. In traditional power systems having uniform structures, all attempts were made to minimize the network loss. In terms of costs, the overall cost of losses is added to other generation and transmission costs and constitute the total operation cost of the network. But in deregulated power systems, any component of the system poses a separate legal character and is therefore independent in terms of income and costs. Thus, determining their share in total network costs including the losses is unavoidable [1]. On the other hand, in deregulated power systems, regardless of

optimizing losses, another serious question that crops up is how much of the total cost of losses should each of the power market players pay. In the pool-based electricity market, loss allocation helps to recognize the share of each generation or consumption unit within the total network losses. This helps the ISO to receive the losses' costs from each market participant and return it to the generation companies (GenCos) [2].

In markets that are based on bilateral or multilateral transactions, the losses of each transaction should be specified in the transaction content and its support source should be determined. In spite of the immense importance of loss allocation to the participants, technically and economically, due to complexity, nonlinear nature and high dependence of loss function on different variables, no comprehensive and precise method which can be practically employed has been presented so far. Owing to the significance of this issue, various methods have been published in papers and most of them have used simple

\* Corresponding author: Amin Moradkhani  
Address: Faculty of Engineering, Ilam University, Ilam, Iran  
Postal Box: 69315-516  
Email: a.moradkhani@ilam.ac.ir

assumptions. In the pro rata method [3], which is the most popular one, the loss is allocated to each generator or load, depending on its power injection to the network rather than total network power injection. In fact, this method does not consider their location or network topology. So a remote generator or load that causes more power losses is treated the same as other near network players.

The proportional sharing principle is based on a non-provable or disprovable theorem that assumes that power inflow is proportionally shared with the outflow power at each network bus [4]-[5]. This method uses an additional assumption that 50% of the losses of each branch can be allocated to their sending and ending nodes.

References [6] suggest a radial equivalent network for a transmission system where each generator may have an individual connection to all loads. In this way it is possible to separate system loss. However, the total losses may not be equal to the real system and may also be too complicated for real power systems. References [7]-[8]-[9] trace the losses back from the network branch to the load. These strategies generally involve an algorithm to determine how the losses attributed to generators/loads accumulate as one traverse through the network. Either the algorithm allows loss attribution to be specified according to a user-defined formula, or a loss sharing formula is implicitly included.

The cooperative game theory was utilized [10] to allocate transmission costs to wheeling transactions. A method, based on circuit theory, has also been proposed to trace power from either the seller and/or the buyer's point of view [11]. In [12], line power flows are first unbundled into a sum of components, each corresponding to a bilateral transaction. The scheme then proposes ways in which the coupling terms among the components appearing in the line losses can be allocated to individual bilateral transactions. In [13] a process is used whereby individual bilateral transactions are gradually increased along a given variable path. Each bilateral transaction may elect to have its losses supplied by a separate slack generator.

In [9] starting from an AC load flow solution, the contributions of all generators to the flow in each circuit are evaluated and the same proportion is used to share circuit losses among them. The Z-bus loss allocation uses the total system loss formula and tries to write it as the summation of each bus complex current injection [14].

Finally, in [15] a loss allocation method has been introduced in bilateral markets. In order to apply the loss allocation to transactions, this method uses the branch current circuit equations. In this reference, each transaction contains a sending bus (seller) and several receiving buses (buyer). The loss allocation problem in multi-area transmission networks is studied in [16].

In paper [17], we have proposed a method by which the share of bus  $k$  from the total losses of the network can be calculated in a pool-based electricity market. In [18], a simple method is proposed to trace the active power flows. In Ref [18] for a given operating point obtained by the AC load flow, the so-called injection-bus and extraction bus matrices are formed by the system. By using the properties of these matrices, three other matrices are derived, which allow the active power production of the generators to be expressed in terms of the active power consumption of the loads, and vice versa. These matrices provide the opportunity to determine the contribution of each generator towards the active loss of each branch and active power consumed by each load as well as to allocate the active loss of each branch to each load. Ref [19] presents a fast artificial intelligence-based incremental transmission loss allocation (ITLA) algorithm for determining the loss quota of any transaction and participant entity in open access environments. As a feature selection technique, the decision tree (DT) method is applied in order to define the transactions with inconsiderable impact on the loss quota of each market participant. Then, using only the effective transactions, an artificial neural network (ANN) is trained to estimate the loss quota of each transaction in the market.

In this paper using transmission line loss equations with respect to bus injected currents, the share of each bus from the mentioned transmission line losses has been determined. Then, this method has been applied to the total network transmission lines and the share of each bus from the total losses while considering the available transaction has been acquired. The proposed method is based on the main network relations and no simplifying assumptions have been used. In the next sections, the proposed method is studied on a typical network.

## **2. Bilateral or multilateral transaction-based markets**

In bilateral or multilateral transaction-based markets, the consumer is treated as a buyer, in order to receive the required power contracts

from the generation companies (GenCos). Similarly, a GenCo is considered as a seller of power, contracts to the consumers.

For each  $m$  ( $m=1, 2, \dots, M$ ) a  $T^m$  transaction includes a set of selling entities ( $S^m$ ), buying entities, the losses compensation portion ( $l^m$ ) and the MW amount of the each transaction which is as follows:

$$T^m = \{t^m, S^m, B^m, l^m\} \quad (1)$$

Where:

$$S^m = \{(s_i^m, \alpha_i^m), i=1,2,\dots, N_s^m\} \quad (2)$$

$$B^m = \{(b_j^m, \beta_j^m), j=1,2,\dots, N_b^m\} \quad (3)$$

$$l^m = \frac{P_{loss}^m}{t^m} \quad (4)$$

For each transaction with the allocated loss of  $P_{loss}^m$ , the seller bus  $s_i^m$  provides the  $\alpha_i^m$  portion of the total transaction amount. On the other hand, the buyer bus  $b_j^m$  receives the  $\beta_j^m$  portion of the total transaction amount. The  $l^m$  portion of each transaction with  $t^m$  value is the portion of the system loss in slack bus which is allocated to that transaction.

Before solving the problem of loss allocation the value of  $l^m$  is not appointed. Hence, the iteration process is inevitable to find it. The injected power in system buses with respect to the available transactions in the market can be written as follows:

$$P_k = \sum_{m=1}^M \delta_k^m t^m \quad (5)$$

Where  $P_k$  and  $t^m$  are the injected power to bus  $k$  and the traded off MW value of  $m$  transaction respectively. For network buses, the expected slack bus,  $\delta_k^m$  as follows:

$$\delta_k^m = \begin{cases} \alpha_i^m & \text{if } s_i^m = h, i=1,2,\dots, N_s^m \\ -\beta_j^m & \text{if } b_j^m = h, j=1,2,\dots, N_b^m \\ \alpha_i^m - \beta_j^m & \text{if } s_i^m = b_j^m = h, i=1,2,\dots, N_s^m, j=1,2,\dots, N_b^m \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

For slack bus we have:

$$\delta_k^m = \begin{cases} \alpha_i^m + l^m & \text{if } s_i^m = h, i=1,2,\dots, N_s^m \\ l^m - \beta_j^m & \text{if } b_j^m = h, j=1,2,\dots, N_b^m \\ \alpha_i^m - \beta_j^m + l^m & \text{if } s_i^m = b_j^m = h, i=1,2,\dots, N_s^m, j=1,2,\dots, N_b^m \\ l^m & \text{otherwise} \end{cases} \quad (7)$$

For example, consider a network with  $M$  transactions. For bus  $k$ , the injected power can be written as:

$$P_k = \sum_{m=1}^M \delta_k^m t^m = \delta_k^1 t^1 + \delta_k^2 t^2 + \delta_k^3 t^3 + \dots + \delta_k^M t^M \quad (8)$$

The above equation is a linear relation for each transaction so the share of  $h$  transaction from the injected power to bus  $k$  can be formulated as:

$$P_k^h = \delta_k^h t^h \quad (9)$$

A typical 3-bus network has been shown in Fig.1. This network has two generators with a production of 900 MW. Also, the two available transactions between the sellers and buyers of this network are as the following:

$$T^1 = \{500, \{(1,100\%), \{(1,80\%), (2,20\%), l^1\} \\ T^2 = \{400, \{(3,100\%), \{(1,25\%), (2,50\%), (3,25\%), l^2\} \quad (10)$$

In this network  $T^1$  transaction has a value of 500 MW and considers bus 1 as the seller, which provides the total required power. The available loads in bus 1 and bus 2 buy 80 and 20 percentage of the total generated power.

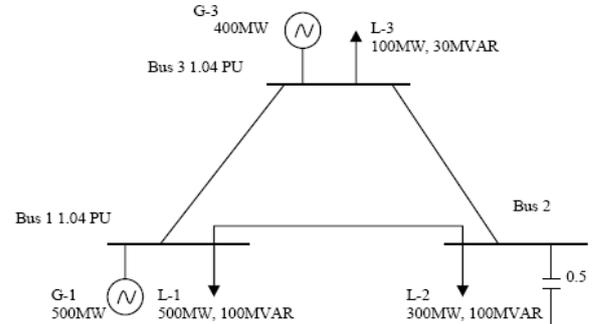


Fig.1. A typical 3-bus network

Also  $T^2$  transaction with a value of 400 MW and considering bus 3 as the seller provides the total required power. The available loads in bus 1 and bus 3 buy 25 and 50 percentage of this total generated power. According to the transactions, the nodal injection for bus 1 is as given below:

$$\sum_{m=1}^2 \delta_1^m t^m = (100\% - 80\%)t^1 + (-25\%)t^2 = 0.2t^1 - 0.25t^2 \quad (11)$$

In this equation,  $t^1$  is the measure of the first transaction's power and  $t^2$  is the second transaction's power.

### 3. The proposed strategy for transmission loss allocation in bilateral or multilateral transaction-based markets

In power networks, the total loss is due to the power flows in the transmission lines. In fact, the total loss is the sum of the losses of all transmission lines. Suppose the power flow results, state estimation for an n-bus network with its topology are available. Subsequently,

$$Ploss^k = \sum_{i=1}^n \sum_{j=1}^n 0.5 \times P_{Lij}^k \Rightarrow$$

$$Ploss^k = \sum_{i=1}^n \sum_{j=1}^n -0.5 \times G_{ij} \times$$

$$\left\{ \begin{aligned} & |(z_{ik} - z_{jk})I_k|^2 + 2 \times \Re((z_{ik} - z_{jk})I_k) \times \\ & \sum_{m=1, \neq k}^n \Re((z_{im} - z_{jm})I_m) \times \frac{|\Re((z_{ik} - z_{jk})I_k)|}{|\Re((z_{ik} - z_{jk})I_k) + |\Re((z_{im} - z_{jm})I_m)|} + \\ & 2 \times \Im((z_{ik} - z_{jk})I_k) \times \sum_{m=1, \neq k}^n \Im((z_{im} - z_{jm})I_m) \\ & \times \frac{|\Im((z_{ik} - z_{jk})I_k)|}{|\Im((z_{ik} - z_{jk})I_k) + |\Im((z_{im} - z_{jm})I_m)|} \end{aligned} \right\} \quad (12)$$

On the other hand, the total losses of a transaction are equal to the sum of the share of transaction participants in network buses. Also, network buses may be available in other transactions. Using the Eq.(9), the share of transaction m from the injected power to bus k can be calculated. So, the share of transaction m in producing the losses in bus k using the Eq.(9) and principle of proportional division is as follows:

$$Ploss^k(\text{transaction } m) = Ploss^k \times \frac{|\delta_k^m t^m|}{\sum_{m=1}^M |\delta_k^m t^m|} \quad (13)$$

Now, the total losses of the transaction m are the sum of its participant losses in network buses:

$$Ploss(\text{transaction } m) = \sum_{k=1}^n Ploss^k \times \frac{|\delta_k^m t^m|}{\sum_{m=1}^M |\delta_k^m t^m|} \quad (14)$$

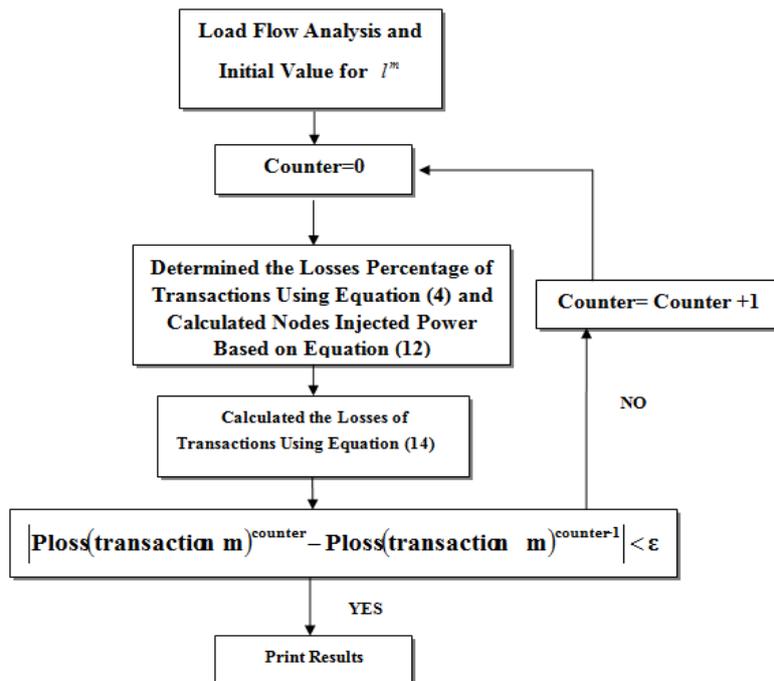


Fig. 2. The flowchart of the proposed method

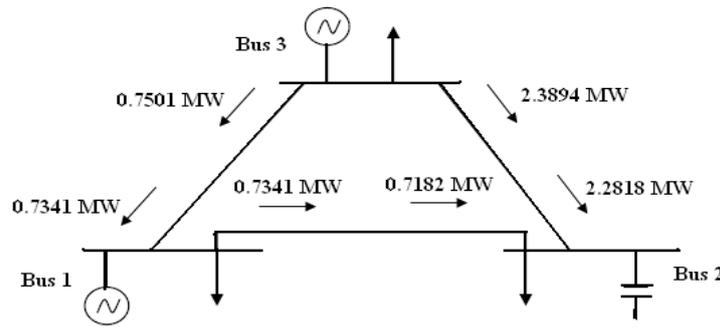


Fig. 3. A typical 3-bus system and the power flow results under transaction 3

As mentioned before, owing to the absence of  $l^m$  value, an iteration process should be done to obtain its value. Figure 2 shows a flowchart of the proposed method.

#### 4. Numerical results

A 3-bus system for loss allocation to the available transactions has been shown in Fig.3.

Also, the required data and the results of power flow of the mentioned system are illustrated in Tables 1 and 2, respectively. Two different cases are listed. Note that the power flow solution and loss allocation results in Table 2 and hence the total system losses are the same for all five cases.

Case 1

$$T^1 = \{500, \{(1,100\%), \{(1,40\%), (2,60\%), l^1\}$$

$$T^2 = \{400, \{(3,100\%), \{(1,75\%), (3,25\%), l^2\}$$

Case 2

$$T^1 = \{400, \{(1,50\%), (3,50\%), \{(1,100\%), l^1\}$$

$$T^2 = \{400, \{(1,50\%), (3,50\%), \{(1,25\%), (2,75\%), l^2\}$$

$$T^3 = \{100, \{(1,100\%), \{(3,100\%), l^3\}$$

Table 3 shows the results of the loss pertaining to the cases. The negative loss allocated to them can be justified by using the principle of mutual and parallel flow effect. Figure 4 shows the flow of transmission lines with respect to the available transactions. Consider case 3 in which 100 MW is transmitted from bus 1 to bus 3. As can be seen, this transaction leads to the production of flows in a direction opposite to that of dominant flows and a reduction in the total losses of the network. So, the negative loss allocation has been appointed to transaction 3.

Table 1. The data of the typical 3-bus system

Line From Bus to Bus	R (%)	X (%)	B (%)
1-2	0.02	0.08	0.0025
1-3	0.03	0.12	0.0025
2-3	0.02	0.06	0.0025

Table 2. The results of power flow in typical 3-bus system

Bus. No	Voltage	Angle	PG(MW)	QG(MVAr)	PD(MW)	QD(MVAr)
1	1.04	-5.0188	500	176.81	500	100
2	0.984	-7.6843	0.00	50.000	300	100
3	1.04	0.0000	413.93	46.5300	100	30
Total Sum			913.93	273.24	900	230

Table 3. The results of loss allocation to transactions

Allocated loss	Case 1	Case 2
T <sup>1</sup> Transaction	6.4651	4.875
T <sup>2</sup> Transaction	7.4746	11.4801
T <sup>3</sup> Transaction	0.000	-2.4375

## 5. Conclusion

In this paper, the problem of loss allocation in power systems has been studied and a new method to combat loss allocation in contract-based markets has been proposed. First the loss relations of a transmission line with respect to bus injected currents and the share of each bus from the mentioned transmission line losses was determined. Then, this method was applied to the entire network's transmission lines and the share of each bus from the total losses was determined. The proposed method was based on the main network relations and no simplifying assumption was used. The proposed method was tested on a typical 3-bus system. The results showed the precision of the method in loss allocation. In fact, the simplicity of the method used the main relations of the network and injected power into the buses. This propelled the market participants towards the loss reduction. The location of the generators and the loads in the network, compatibility with the power flow results and appropriate algorithm are the main features of the proposed method.

## References

- [1] Ilic M., Galiana F., Fink L., Power Systems Restructuring: Engineering and Economics, Norwell, MA, Kluwer (1998).
- [2] Sheblé G. B., Computational Auction Mechanisms for Restructured Power Industry Operation, Norwell, MA, Kluwer (1999).
- [3] Conejo A.J., Arroyo J.M., Alguacil N., Guijarro A.L., Transmission Loss Allocation, A Comparison of Different Practical Algorithms, IEEE Transaction on Power Systems (2002) 17(3).
- [4] Bialek J. W., Tracing the Flow of Electricity, IEE Proceedings - Generation, Transmission and Distribution (1996) 143: 313-320
- [5] Kirschen D., Strbac G., Tracing Active and Reactive Power between Generators and Loads Using Real and Imaginary Currents, IEEE Transactions on Power Systems (1999) 14:1312-1319.
- [6] Rau N. S., Radial Equivalents to Map Networks to Market Formats, IEEE Transactions on Power Systems (2001) 16(4): 856-861.
- [7] Bialek J.W., Tracing the Flow of Electricity, IEE Proceedings - Generation, Transmission and Distribution (1996) 143: 313-320.
- [8] Macqueen C. N., M.R. Irving, An Algorithm for the Allocation of Distribution System Demand and Energy Losses, IEEE Transactions on Power Systems (1996)11 (1):147-156
- [9] Kirschen D., Allan R., Strbac G., Contributions of Individual Generators to Loads and Flows, IEEE Transactions on Power Systems (1997)12 (1): 52-60
- [10] Tsukamoto Y., Iyoda I., Allocation of Fixed Transmission Cost to Wheeling Transactions by Cooperative Game Theory, IEEE Transactions on Power Systems (1996)11 (2): 620-627
- [11] Chang Y.C., Lu C. N., An Electricity Tracing Method with Application to Power Loss Allocation, International Journal of Electrical Power & Energy Systems (2000) 23 (1): 13-17
- [12] Gomez Exposito A., Riquelme Santos J. M., Gonzalez Garcia T., Ruiz Velasco E. A., Fair Allocation of Transmission Power Losses, IEEE Transactions on Power Systems (2000) 15(1): 184-188.
- [13] Galiana F. D., Phelan M., Allocation of Transmission Losses to Bilateral Contracts in a Competitive Environment, IEEE Transactions on Power Systems (2000) 15(1):143-150.
- [14] Conejo A. J., Z-Bus Loss Allocation, IEEE Transactions on Power Systems (2001) 16: 105-110.
- [15] Adsoongnoen C., Ongsakul W., Maurer C., Haubrich H., Transmission Pricing Using the Exact Power and Loss Allocation Method for Bilateral Contracts in a Deregulated Electricity Supply Industry, European Transactions on Electrical Power (2007)17:240-254 DOI: 10.1002/etep.131.
- [16] Kazemi A., Jadid S., Andami H., A Circuit Based Method for Multiarea Transmission Networks Loss Allocation, European Transactions on Electrical Power (2008) 18:753-766 DOI: 10.1002/etep.225.
- [17] Aazami R., Monsef H., A Directional-Based Branches Current Method for Transmission Loss Allocation in the Pool-Based Electricity Market, Energy Equipment and Systems (2016)4(2): 177-187. DOI: 10.22059/ees.2016.23036.
- [18] Enshaee P., Enshaee A., Approach to Evaluate Active Loss Contributions for Transmission Systems, IET Science, Measurement & Technology (2016) 10(5): 456-466.
- [19] Kargarian A., Raoofat M., Mohammadi M., Artificial Intelligence-Based Loss Allocation Algorithm in Open Access Environments, Journal of Energy Engineering (2014)140 (2): 1-9.