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A mixed integer nonlinear programming model for optimizing a gas pipeline transmission linear network

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ABSTRACT

The technical equipment developed and used in both installation and operation processes in refineries, oil and gas pipelines, and gas booster stations has always been expensive. Hence, managers at different organizational levels are keen to find methods to control and reduce these costs. Generally speaking, the operators in a gas booster station choose the operating devices without considering the related costs. This research presents a mixed integer nonlinear programming model designed to minimize the operational costs of gas booster stations in a main pipeline distribution network. The goal is to optimize the choice of operating devices in these stations to minimize costs while still meeting customer demands. Turbo compressors are chosen as the operating devices and the operational costs are fuel, maintenance, start-up, and penalty costs. However, the significance indexes of these costs are valued differently by the three expert managers: the executive officer, operating head, and the overhaul repairing director. Consequently, the analytical hierarchy process (AHP) method is used to calculate the overall weights of costs, and a gas transmission company in the north of Iran is considered as a case study. The model can minimize the total cost, when compared to the selections of ten experienced operators; however, the absolute weights of choosing measures and the essence of the objective function under study mean that an operator choice exists that would represent the optimum selection of turbo compressors.

1. Introduction

One of the most noticeable features of the distribution and control center of a main gas pipeline network in any country, apart from the development and growth of the main energy production network, is how the production units (e.g., refineries and distribution units) are run.

The control and distribution center is responsible for managing and directing the main gas supply, as well as balancing the consumption at different points, in a process known as dispatching management. The dispatcher's demand due to a gas booster station's customer is fulfilled by control operators in every station who turn on/off the turbo compressors. In the present study, the compressor driver is a gas turbine, which is usually used to provide the coupled torque power needed to drive a rotary machine.

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Generally, the selection of turbo compressors for use in service is done in an empirical manner by the operators, without taking into account the relevant costs. Lastly, the input and output of every gas booster station in a linear network can have many data points; therefore, an online information system bank is useful to permit the best decision making by the dispatcher.

Turbo compressors can have a number of different associated costs, which can be modeled mathematically. Thus a cost benefit analysis program seems to need to execute them. The costs considered here are four types: fuel, repair, start-up, and penalty costs. The aim of the present research was to formulate and minimize these four costs by subjecting them to several constraints. Centrifugal compressors, as rotary machines, should be operated dynamically to perform at steady state conditions. Steady state performance is strongly correlated with the gas flow rate passing through a compressor, as well as with the energy pressure that the compressor can force into the gas. The constraints are shaped as a way of controlling the compressors under steady state conditions. A second aim was to control the service time of the compressors assigned for running, as well as to balance the working points of two or more machines operating in a linear gas transmission.

This paper also provides a review of several important associated studies, as well as the methodology for solving the problem of optimization. The input parameters of the model are classified precisely into several definite elements. Model structuring is described in a later section, and a case study of the model is presented to compare the numerical analyses and to validate the model's contribution.

Nomenclature

A_i,B_i,C_i	Regression coefficient
CCI	cumulative cost index
f	Friction coefficient
F_C	Consumed fuel capacity,m^3/h
HTS	compressor operation time,h
H_P	Polytropic head, m
HV	heating value,kj/m^3
HS	compressor hot start time,h
i	Compressor counter index
k	Polytropic index
k_{st1}	Number of compressors running at station 1
$\dot{m}$	Mass flow rate, kg/s;

P_C	penalty cost
P_j	Output pressure of the first stage, bar
P_l	Output pressure of the second stage, bar
P_{suc}	inlet pressure, bar
P_{dis}	outlet pressure, bar
Q_{ac}	Actual volumetric flow rate, m^3/h
Q_S	Standard volumetric flowrate, mm^3/d
Q_{st}	Compressor gas flowrate , SCMM
R	Universal gas constant, kj/k.mol.k
S	Compressor speed(rpm)
S_C	Start up cost
SFC	Specific fuel consumed, m^3/s
SST	Gas station service time , h
T_{suc}	Station inlet temperature, c^o
W_P	Compressor power, kw
X	Gas station counter index
Z	Gas Compressibility factor
α	Mechanical power loss coefficient
ρ	Gas density, kg/m^3
η_M	Mechanical efficiency
η_P	Polytropic efficiency
η_T	Heating efficiency

2. Literature review

Mathematical programming models are widely used as decision-making tools to determine the optimization patterns in different fields of management and engineering sciences. In industries related to natural gas, integer linear and nonlinear programming models are frequently applied to model gas extraction from wells, distribution processes, transmission, and the location of supply chain contractors. For example, Martin et al. have used mixed integer linear programming to optimize the gas transmission network in Germany. This model structured the gas transmission network as a graph that included pipelines, operational apparatuses, and the line break valves. Its objective function was to minimize the fuel costs of the compressor drivers (the driver can be a gas turbine). The constraints consisted of network pressure and fuel consumption, which were both limited by upper and lower bounds [1]. Borraz-Sánchez and Hoagland formulized a gas pipeline transmission network mathematically as a graph and presented a method based on a

dynamic programming technique to minimize the operation costs, which only included the turbo compressor fuel costs. The gas flow rates in every arc and the output pressure of each station were the decision variables [2]. By contrast, Gopal, Zimmer, and Carter each used dynamic integer programming to solve the issue of minimizing the costs associated with a gas transmission network. The first two researchers restricted the model to a linear network, comprising compressors and the line break valves,while Carter applied it to a cyclic network[3,4,5]. Another study by Dewolf et al. formulized the natural gas distribution challenges by linear programming. Their model minimized the costs under the nonlinear relations that connected the potential pressure energy and the volumetric flowrate[6]. El-Mahdy et al. presented a mathematical programming model to optimize the natural gas transmission network where the objective function was to minimize the costs needed for network construction. The network topography was assumed to be determined in the studied problem, and the pipeline optimum diameter was calculated using a genetic algorithm, while considering two categories of soft and hard restrictions[7]. A gradient research technique was used by Ryan and Parcel to minimize fuel costs in a gas transmission network[8]. Chopman and Abbaspour from Kansas University studied the optimization of gas station functionality using a nonlinear programming method[9].

The practical scope of mixed integer nonlinear programming has also enhanced the study of natural gas purchasing and transmission. For example, Matourana et al. used this mathematical model as a decision-making system to purchase natural gas in a supply chain of local companies in Chile, with the objective function of minimizing the costs to all companies [10]. Cost optimization in a natural gas supply chain was also formulized with mixed integer linear programming by Levary et al. [11]. Chin presented a methodology that used linear programming as a decision-making support system to solve the scaled seasonal demands of customers [12]. Edgar et al. used mathematical modeling to minimize an objective function that included maintenance costs for the gas booster station and pipeline, subject to pressure restrictions within high and low designed pressures and the

pipeline length and diameter.Their model also considered the distance between supply and demand points and they used a branched and bound method and the reduced nonlinear gradient optimization to solve it [13,14]. Cantess, Pert, and Wilson offered a mixed integer programming model for optimizing a gas transmission operation[15,16]. Ruan et al. designed the main pipeline system for a gas distribution network while considering several factors, such as pipeline diameter, size, thickness, pressure, pipeline length, and gas compression ratio, and used a mathematical programming technique in their offered model[17].

The mixed nonlinear model structured by Üster and Dilaveroğlu considered the new network design and operation costs together, with decision variables being parameters like pipeline measurement, compressor station quantity and capacity, the steady state flowrate of network, and the mounting time of the station [18]. The multifunctional optimization model expressed by Kashani and Molaei considered the environmental effect of natural gas,such as the amount of carbondioxide spread into the air, in addition to the operation costs of a natural gas network. Typically, for a double functional model, the boundary answers were obtained using the non-dominated sorting genetic algorithm II [19].

Notably, all these models emphasized minimizing of costs for the gas distribution network without considering the efficiency associated with turbo compressors, which are generally a function of the compressor's flow rate and speed. In particular, the power needed by the compressor to pressurize the gas entering the impeller is inversely related to its polytropic efficiency. Hence, in the present study, the aim was to regressively apply the compressor's polytropic efficiency to the model's objective function. Moreover, the model presented here, in addition to considering the turbo compressor selection, also incorporates the compressor's operational speed according to the steady state conditions and the running time needed for every compressor. Furthermore, this model includes calculations of the intermediate pressures of a three stage centrifugal gas compressor for the given suction and discharge pressures of a proposed gas station.

3.Research methodology

This study examined the use of a mixed integer nonlinear programming method to optimize the process of parallel compressor selection in a linear gas transmission network. This structured model considered the minimizing of the operational costs of turbo compressors situated at every gas station as the objective function, as well as controlling their speeds and operating dynamically at a steady-state condition. Customer demand is considered to determine the input and output pressure and gas flowrate at every gas station. Speed control, steady state functionality conditions, and meeting customer demand are applied to the model as some restrictions. Operational costs are included in the objective function. The model is also compared with the output of ten expert operators. Finally, eight scales are used as the picking up strategies that are weighted by three expert executives and the analytical hierarchy process (AHP) method is used to order these 11 options (the model results and the output of the ten operators) in accordance with the essence of the objective function, which is to minimize total costs. The 8 scales are as follows: minimum total cost, minimum fuel cost, minimum repairing cost index, minimum start-up cost, minimum volumetric percentage of fuel cost, minimum percentage of repair plus penalty costs, minimum percentage of start-up cost, and minimum penalty cost.

4. Mathematical Modeling

The different models mentioned in the literature review mostly involved just the use of input and output pressure factors to estimate the fuel cost of a compressor driver. Others frequently used mathematical techniques to optimize the length or diameter of the gas pipeline. Indeed, we were inspired by these previous studies to determine the entire relevant costs for a linear gas booster station.

The modeling process involves establishing the objective function, the constraints, and decision variables. Thermodynamic formulas are used to determine the consumed fuel cost of a gas turbine. The regression technique is also utilized to formulate the repair and start-up costs of every turbo compressor using the information on money spent during its lifespan.

4.1. Objective function

The model's objective function is to minimize the operational costs of all turbo compressors located in a gas distribution network. In this study, costs related to turbo compressors are classified into the following four categories:
1. Compressor driver fuel cost
2. Maintenance& repair costs
3. Start-up cost
4. Penalty cost

These four costs are brought to their minimums in order to meet the dispatcher demands, including satisfactory flow rate and head pressure.

4.1.1.Fuel cost

The fuel cost function that yields the volumetric fuel consumed (FC) by the centrifugal compressor driver (gas turbine) is generally given as follows:([20],[21]):

$$FC = SFC * W_P \tag{1}$$

$$SFC = {1}/{(\eta_T * \eta_M * HV)} \tag{2}$$

$$W_P = {(\propto * \dot{m})}/{\eta_P} * H_p \tag{3}$$

$$H_P = \tag{4}$$

$$\left(k/{k-1}\right)ZRT_{suc}\left[\left(P_{dis}/P_{suc}\right)^{k-1/k} - 1\right]$$

$$Q_{ac} = ZRT_{suc} * \dot{m}/P_{suc} \tag{5}$$

To determine FC,we move backward from Eqs.(5) to (1).Calculating $\dot{m}$ from Eq.(5) and replacing H_P in relation (3), and then replacing W_P and SFC in relation (1) gives the consumed fuel cost as the final result. Now, by applying the regression model (6) that presents the polytrophic efficiency in terms of the regressor Q/S, the speed of every compressor can be practically entered into the model:([20]):

$$\eta_p = A + B\left(Q/S\right) + C\left(Q/S\right)^2 \tag{6}$$

The coefficients A,B, and C are the regression values, which could be estimated by one hundred performance operational points (*Tsuc,Tdis,Psuc,Pdis,Q,*and*s*), as well as by making use of the direct formula of polytrophic efficiency, as follows:([21])

$$\eta_p = \frac{Tsuc * \left(Pdis/_{Psuc}\right)^{K-1/_K} - Tsuc}{Tdis - Tsuc} \quad (7)$$

The thermal efficiency of the compressor driver (η_T) is usually about 0.3 for gas turbines of under 15 megawatt power[22]. The original formula for calculating the thermal efficiency of the Brayton cycle is more complicated due to several factors for which there is usually no sensor located on a gas turbine. The turbine

no sensitive probe is included for measurement of the internal pressure of a centrifugal compressor. The intermediate pressures(Fig.1) P_l and P_j could be obtained by the following relations.

$$P_j^2 = p_{suc} * p_l, \qquad P_l^2 = p_{dis} * p_j \quad (10)$$

Therefore, the fuel function that gives the consumed volumetric fuel burned to produce the power needed to run a three-stage centrifugal compressor is as follows:

$$FC = \frac{\alpha Q p_{suc}\left(k/_k - 1\right)\left[p_{suc} * \left[\left(p_j/_{p_{suc}}\right)^{k-1/_k} - 1\right] + p_j * \left[\left(p_l/_{p_j}\right)^{k-1/_k} - 1\right] + p_l * \left[\left(p_{dis}/_{p_l}\right)^{k-1/_k} - 1\right]\right]}{\eta_M * \eta_T * HV * \eta_P}$$

$$(11)$$

compressor efficiency (η_c)is an example of these factors.([22])

$$\eta_{cycle}$$
$$= \left(\frac{\eta_t t_f - \dfrac{t_{amb} r_p^{\left(\gamma-1/_\gamma\right)}}{\eta_c}}{t_f - t_{amb} - t_{amb}\left(\dfrac{r_p^{\left(\gamma-1/_\gamma\right)} - 1}{\eta_c}\right)}\right)$$
$$\left(1 - \frac{1}{r_p^{\left(\gamma-1/_\gamma\right)}}\right) \quad (8)$$

Note that the gas flow rate capacity on every compressor, as requested by the dispatcher, is metered in SCMM (million cubic meters standard) per day. Consequently,Eq. (9) would be used to calculate the actual volumetric flow rate in terms of cubic meters per hour.([21])

$$Q_{ac}$$
$$= \frac{(Q_{st}) * 1.013 * (273.15 + T_{suc}) * Z_{suc}}{0.0864 * P_{suc} * 288}$$
$$* 3600 \quad (9)$$

Note also that the Eq.(3) provides the needed compressor power to compress the gas moving across the impeller of a one-stage centrifugal compressor. Therefore, it is necessary to calculate this for three-stage centrifugal compressors. Henceforth, the intermediate internal pressures are needed for a three-stage compressor. Of course, as originally designed,
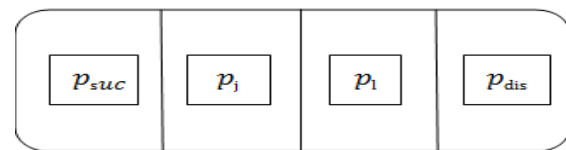

| p_{suc} | p_j | p_l | p_{dis} |

Fig. 1. Internal pressures of a three stage centrifugal gas compressor

The parameter α in Eq.(11) is the corrective mechanical coefficient, which is usually considered as 0.95.

$$CCI_t \quad (12)$$
$$= \frac{\sum_0^t GrossExpenditures + PurchasePrice_0}{PurchasePrice_0}$$

The numerator of CCI is the sum of the net present value of expenditures associated the machine's operational working hours and the denominator is the equipment purchase price. Mathematically, this index is dimensionless.

Now, for a suitable selection of the regression formula of each turbo compressor's repair cost, the cumulative working hours (x) and CCI are respectively considered as the independent and dependent variables. The regression models used to estimate the cumulative cost index (y=cci) are the following: ([23])

$$y = \beta_0 + \beta_1 x + \varepsilon \quad (13)$$
$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \varepsilon \quad (14)$$
$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \varepsilon \quad (15)$$
$$y = \beta_0 + \beta_4 e^x + \varepsilon \quad (16)$$
$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_4 e^x + \varepsilon \quad (17)$$
$$y = \beta_0 + \beta_3 x^3 + \varepsilon \quad (18)$$

$$y = \beta_0 + \beta_1 x + \beta_3 x^3 + \varepsilon \tag{19}$$

$$y = \beta_0 + \beta_1 x + \beta_4 e^x + \varepsilon \tag{20}$$

$$y = \beta_0 + \beta_1 x + \beta_3 x^3 + \beta_4 e^x + \varepsilon \tag{21}$$

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 e^x + \varepsilon \tag{22}$$

$$y = \beta_0 + \beta_2 x^2 + \beta_3 x^3 + \varepsilon \tag{23}$$

$$y = \beta_0 + \beta_2 x^2 + \beta_4 e^x + \varepsilon \tag{24}$$

$$y = \beta_0 + \beta_2 x^2 + \beta_3 x^3 + \beta_4 e^x + \varepsilon \tag{25}$$

$$y = \beta_0 + \beta_3 x^3 + \beta_4 e^x + \varepsilon \tag{26}$$

$$y = \beta_0 + \beta_2 x^2 + \varepsilon \tag{27}$$

In this study, the value of β_0 is always one because, in zero cumulative working hours (x=0), the value of CCI would be one. Some functions of the regressor x also exist, namely x, x^2, x^3, and e^x. These four terms were chosen because they each can describe the monotonically increasing line that defines the CCI in relation to cumulative working hours. The optimum regression model between these fifteen mentioned formulas is selected by first confirming that the p-values of the regression coefficients are statistically significant, and then comparing the R square index among the accepted ones. Finally, based on the regression models with the same R square value, the model with the minimum standard estimated error is the nominee used to explain the cumulative cost index.

4.1.3. Start-up cost

The start-up costs are generally connected to the special electrical equipment that is necessary while the turbo compressor is operating. In this research, the regression method is used to consider the directly proportional relation between these kinds of costs and the compressor speed (S) and its service time hours (HTS):

$$Sc \propto (S*HTS) \tag{28}$$

4.1.4. Penalty cost

The penalty cost is related to the device that is not selected by the model. If a turbo compressor is a candidate for turning off to minimize its total cost, it would pay a penalty cost equal to the sum of the other turbo compressors' fuel costs:

$$PC = ((FC_j * HTS_j) * G_j + (FC_k * HTS_k) * G_K + \dots\dots + (FC_{N-1} * HTS_{N-1}) * G_{N-1} - (HS - (HS/N) * G_i - (HS/N) * G_j - \dots - (HS/N) * G_N)) * (1 - G_i) \tag{29}$$

$$i \neq j, k, \dots\dots, N - 1$$

Now, the objective function for a gas transmission network that includes several gas booster stations is as follows:

MIN:z=

$$\sum_{x=1}^{m} \sum_{i=1}^{n} \left[(FC_{x,i} * HTS_{x,i}) * G_{x,i} + (MC_{x,i}) * G_{x,i} + (SC_{x,i} * HTS_{x,i}) * G_{x,i} + (PC_{x,i}) \right] \tag{30}$$

4.2. Decision variables

Three variables G={0,1}, HTS, and S are used in the model. Variables G, HTS, and S are respectively associated with the compressor's on/off state, service time, and speed.

4.3. Constraints

The behavior of a typical dynamic compressor is excessively affected by factors like gas flow rate, input/output pressures, and gas speeds; therefore, the aim is to control its steady-state performance while logically restricting its service time. However, the dispatcher's demands and the real input parameters (*psuc,pdis,q,tsuc*) must also be entered into the constraints to meet the conditions that the dispatcher desires.

The first constraint restricts the compressor speed between the upper and lower bounds, which, in this case are really the maximum and minimum of the compressor's nominally designed speeds. Operationally, the compressor's minimum speed is a function of the compressor ratio (*pdis* per *psuc*) and the gas flow rate passing through each compressor; however, the compressor's maximum speed must not exceed some predefined major limit at which the gas velocity entering into the compressor's impeller reaches the speed of sound (known as the mach number). This phenomenon is potentially dangerous and could lead to a surge state. These limits are specifically defined for every turbo

compressor. In this research, they are precisely identified as 7200 rpm.

$$S_{min} = \alpha * \left({p_{dis}}/{p_{suc}} \right) + \beta * q \leq s \qquad (31)$$
$$\leq s_{max}$$

Clearly, the coefficients α and β could be evaluated by a regression method just by applying the past operational data for each turbo compressor. In fact, the parameter S_{min} can be regressively estimated in terms of ${p_{dis}}/{p_{suc}}$ and q.

The second constraint concerns the dynamic behavior of the compressor at a steady state; i.e., avoiding a surge condition. The vital factor that determines the compressor's performance at steady or non-steady state conditions is shown by Q/S (flowrate/ speed). Moreover, this element actually relates the parameters of q (flow rate) and s (compressor's speed) to the effective polytropic efficiency of each compressor:

$$1 \leq {q}/{s} < 2.2 \qquad (32)$$

The third constraint involves the process of dividing the total gas flow rate passing through each station onto paralleled compressors so that it is balanced on running compressors while one device remains off. This pattern is formulated as Eq.(32). Technically, for a gas booster station, the aim is to have at least one turbo compressor on standby. The scrubbers installed in each station also normally have a transmission capacity that covers all the turbo compressors minus one.

$$\sum_{i=1}^{n-1} q_i * g_i = q \qquad (33)$$

The fourth constraint is structured in order to regulate the compressor's performance hours. The aim is to have the mean of the differential operating hours of the compressors located in two stations X,Y not to exceed a particular limit(this limit will be explained in the next parts). Actually, with this constraint, an attempt is made to maintain a balance of the overall equivalent operating hours of all compressors due to overhaul service time. This means that the time differences between the compressors in a linear network are rather close to each other to allow scheduling of overhaul service time.

$$\left| {\sum_i^n g_{x,i} * hts_{x,i}}/{\sum_i^n g_{x,i}} \right.$$
$$\left. - {\sum_j^m g_{y,j} * hts_{y,j}}/{\sum_j^m g_{y,j}} \right| \qquad (34)$$

$$\leq MAX(sst /number of units selected)$$

The fifth restriction is applied to the model to control the operating hours of selected compressors in each gas transmission station. The aim of this constraint is to close the operating time service between the compressors. Technically speaking, internal combustion motors, just like the turbo compressors studied here, should have uniformly timed service during their lifespan.

$$\left| ht_i - hts_j \right| * g_i * g_j$$
$$\leq (\Delta h - \beta * \Delta h) * g_i \qquad (35)$$
$$* g_j \,, 0 \leq \beta \leq 1$$

The sixth constraint estimates the total service time of all selected running compressors at every gas booster station. In other words, it designates the determined service time of an individual station. This restriction is constructed according to the hydraulic and thermodynamic relations of pipeline between the two gas stations.

$$\sum_{i=1}^{n} g_i * \left({q_{st}}/{q_i} \right) * hts_i \qquad (36)$$
$$= station service time$$

The station service time is empirically calculated in terms of the input/output pressures, the gas flow rates passing between two stations, the pipeline capacity between them, the friction coefficient of the internal surface of the pipeline, and the inlet/outlet temperature. The gas velocity (u)in a pipeline depends on several parameters, such as gas pressure, temperature, pipeline internal diameter,gas flow rate, compressibility factor(z), base pressure (pb), and base temperature(tb). Using the fan laws and the technical behavior of the gas flow rate, the station service time is finally calculated while satisfying the parameters for the input and output of each gas station (Fig.2).

$$\text{station service time} = \frac{k_{st1} * \left(S_{minst1}/S_{maxst1}\right) * (p_b) * f * p_{dis1} * q_{st2} * t_{suc2} * v_{bp}}{\left(p_{dis1}^2 - p_{suc1}^2\right) * q_{st1}^2 * t_{dis1}} \tag{37}$$
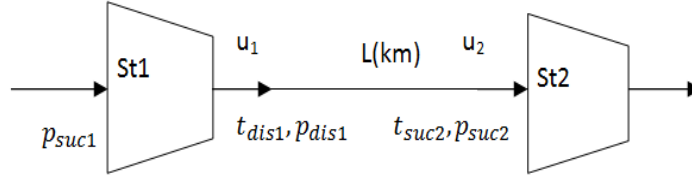

Fig.2. Schematic layout of the two linear compressor stations

where V_b is the volumetric capacity of the pipeline in terms of cubic meters at standard conditions[24]. D and L are also, respectively, the internal diameter of pipeline in millimeters and its length in kilometers.The values of *psuc2* and *Tsuc2* are the real inputs for the downstream compressor station 2. Actually, the values of *tdis1* and *pdis1*(upstream compressor station outputs) are decreased due to the internal surface friction of the pipeline; this is a natural phenomenon that occurs in every gas pipeline.

$$\begin{aligned} V_b \\ = 7.855 \\ * 10^{-4} \left(T_b/P_b\right) \left(P_{avg}/Z_{avg}T_{avg}\right)(D^2L) \end{aligned} \tag{38}$$

The seventh constraint is used to control the speeds of similar compressors located at each station. The usual experience, when two or more compressors are running at each station,is that their speed difference is not greater than 150RPM, although clearly the speed difference can be every value based on the surge line of every compressor. However, the paralleled layout of compressors creates the same differential pressure among the all the running compressors, regardless of their speeds.

$$\left|s_i - s_j\right| * g_i * g_j \leq speeddiff \tag{39}$$

The dispatcher usually tries to keep the running compressor's speed close to that needed for full pressurization of the pipeline and to satisfy the input gas flow rate of each station. Hence, the aim is to ensure that the mean differential speed selected for the two stations does not exceed a particular limit (Δs). Consequently, the eighth restriction is as follows:

$$\left|\sum_i^n g_{x,i} * s_{x,i}\middle/\sum_i^n g_{x,i}\right. \tag{40}$$
$$\left.-\sum_j^m g_{y,j} * s_{y,j}\middle/\sum_j^m g_{y,j}\right|$$
$$\leq (\Delta s - \beta * \Delta s) , 0 \leq \beta \leq 1$$

According to the performance map of centrifugal compressors (Fig.3), the operational points of the compressors should be controlled between the surge and stonewall limits to dynamically functioning. The operators also try to have these operational points very close to each other when two or more machines are running. This is because the best process control would be achieved due to the same divided load. Here, the straight distance difference among the mentioned points is considered as ε:

$$\left|s_i - s_j\right| * g_i * g_j \leq \varepsilon * s_i * s_j \tag{41}$$

5.Model input parameters

The control room operators have generally one degree of freedom to satisfy the dispatcher's demand, and that is to regulate the compressor speed. In this study, the input parameters are as follows:

- Station suction pressure(P_{suc})
- Station discharge pressure(P_{dis})
- Station gas flow rate (Q_{ac})
- Station inlet temperature(T_{suc})
- Minimum working hours of every compressor after hot start (HS)
- Turbine thermal efficiency (η_t)
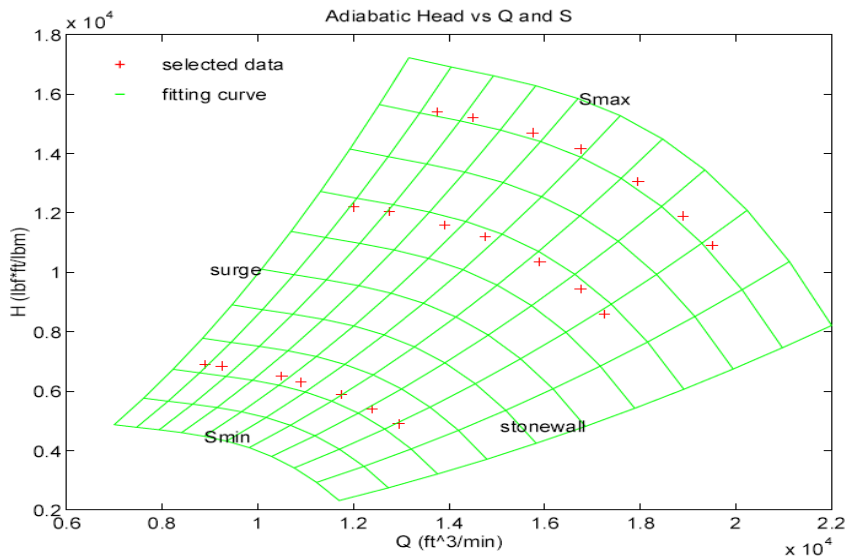- Turbine mechanical efficiency (η_m)

Fig. 3. A typical centrifugal compressor map (Wu, 1999)

- Heating value of turbine fuel (HV)
- Polytrophic index of compressed gas (K)
- Hundred operating points to estimate the regression coefficients of η_p and S_{min}
- Points indicating the repair costs corresponding to cumulative operating hours to estimate the repair cost index at a period of time
- Maximum differential mean of the compressor speeds(compressors of two stations) and differential operating hours of the compressors at each station (Δs)

It should be necessarily mentioned that the input parameters of each station (suction pressure & temperature) are the real numbers gauged by the instrumental devices mounted on station's pipelines. And also the discharge pressure and the gas flow rate are the dispatcher's demands.

6. Case study

In this part, the model is applied to a real gas transmission network and its results are compared with ten expert operator selections. Note that the compressors in this study are all assumed to be turned off at first. Iran's gas transmission operation network no.9, composed of 4 gas booster stations with paralleled turbo compressors, is the real case used to evaluate the model (Fig.4).

The technical information of the studied gas pipeline is listed in the Tables (1) and (2), and some input data of the model are shown in Table (3).

Table 1.Compressor station characteristics

Comp. min &max speed(rpm)	Maxflowrate (m³/h)	Arrangement	Compressor quantity	Station name
5450,7400	11441	parallel	3	Ghalejigh
5450,7400	11441	parallel	3	Neka
5450,7400	11441	parallel	2	Noor
5200,7400	5733	parallel	4	Ramsar

Table 2.Pipeline characteristics

Friction coefficient	Absolute roughness (mm)	Thickness (cm)	Int.diameter (inch)	Ext.diameter (inch)	Material	
0.011	0.045	1.5	28.8	30	Welded steel	Pipeline tech scale

Table 3. .Model input data

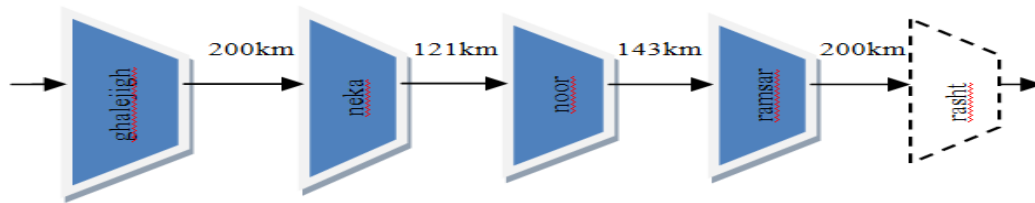| ε	Turbine η_M	Turbine η_T	Z	k	Δh	Δs	Speed diff.	HS	HV (kJ/m³)
0.008	0.95	0.3	0.95	1.28	20	1000	150	10	36000

Fig.4. Linear network of Iran's gas transmission no.9

Note that if a gas flow rate demanded by dispatcher exceeds the maximum flowrate of a compressor, then the actual gas flowrate should be divided between the compressors and used into the model.

The dispatcher's demand as a customer of the network under study is listed in Table (4).

In this section, the best estimation of the parameter CCI between the 15 formulas for repair cost is determined by a typical regression of the data of one of the Neka turbo compressors listed in Table (5). The assumptions listed below are also used to enter the data into the SPSS^R software. The level of significance is assumed to be 0.05.

$$t1=x \;,\; t2=x^2 \;,\; t3=x^3 \;,\; t4=e^x \qquad (42)$$

In order to start writing the model code in the LINGO programming spread sheet, two indexes have been deployed. The first one (X) enumerates the gas booster stations and the second (i) enumerates the turbo compressors situated at each station.

Importing the input parameters into the model and coding it in LINGO version 9 prepares it for solving at a machine equipped with dual core Intel chips,1.22 RAM,2.66GHZ, and GB,E7300. After about 14 seconds, the model outputs are gathered; these are listed in Table (6).

Table 4. Dispatcher's demand

$T_{suc}(c^o)$	$Q_S(MM^3/d)$	Output press.(bar)	Input press.(bar)	Station name
26	16	52	43	Ghalejigh
25	14	53	45	Neka
27	11	46	40	Noor
27	10	42	36	Ramsar

Table 5. The regression data of one of the Neka compressors

Regression model	Coeff.	t	sig	Adj.r.square	Std. error of the estimate
1	constant=1.001	1511.129	.000	.996	.001682
	t1=.003	73.530	.000		
2	constant=1.000	1079.475	.000	.996	.001582
	t1=.004	19.890	.000		
	t2=-1.349E-005	-2.000	.058		
3	constant=1.001	854.421	.000	.996	.001556
	t1=.003	7.121	.000		
	t2=3.891E-005	.963	.347		

	t3=-1.419E-006	-1.315	.203		
4	constant=1.037	225.452	.000	.325	.020793
	t4=2.417E-012	3.540	.002		
	constant=1.000	1010.239	.000		
5	t4=7.684E-015	.088	.930	.996	.001619
	t1=.004	16.870	.000		
	t2=-1.400E-005	-1.562	.133		
6	constant=1.022	367.389	.000	.842	.010049
	t3=4.668E-006	11.363	.000		
	constant=1.000	1221.850	.000		
7	t3=-3.936E-007	-2.225	.037	.996	.001554
	t1=.003	30.660	.000		
	constant=1.001	1431.891	.000		
8	t1=.003	59.491	.000	.996	.001671
	t4=-7.886E-014	-1.142	.266		
	constant=1.000	1152.381	.000		
9	t1=.003	26.582	.000	.996	.001581
	t4=4.646E-014	.499	.623		
	t3=-4.848E-007	-1.891	.073		
	constant=1.001	844.515	.000		
	t1=.003	5.972	.000		
10	t4=1.400E-013	1.284	.214	.996	.001533
	t3=-2.606E-006	-1.850	.079		
	t2=7.354E-005	1.530	.142		
11	constant=1.008	893.215	.000	.988	.002810
	t3=-8.423E-006	-10.506	.000		
	t2=.000	16.498	.000		
	constant=1.014	524.013	.000		
12	t2=.000	15.848	.000	.943	.006034
	t4=-7.259E-013	-2.590	.017		
	constant=1.007	967.098	.000		
13	t2=.000	16.462	.000	.990	.002496
	t4=4.203E-013	2.624	.016		
	t3=-1.020E-005	-10.372	.000		
	constant=1.020	388.210	.000		
14	t4=-1.155E-012	-2.469	.022	.871	.009093
	t3=5.776E-006	9.914	.000		
15	constant=1.016	493.233	.000	.929	.006741
	t2=.000	17.750	.000		

In order to start writing the model code in the LINGO programming spread sheet, two indexes have been deployed. The first one (X) enumerates the gas booster stations and the second (i) enumerates the turbo compressors situated at each station.

Importing the input parameters into the model and coding it in LINGO version 9 prepares it for solving at a machine equipped with dual core Intel chips,1.22 RAM,2.66GHZ, and GB,E7300. After about 14 seconds, the model outputs are gathered; these are listed in Table (6).

The model output is now compared with the ten expert and skillful operators' selections. They are invited to express their points of view about the turbo compressors that are nominated to be turned on according to the information presented in Table (4). The operators' selections result in some costs due to the same objective function of the main model. These costs are as follows:

- Total cost (summation of four costs below that is the main objective function)
- Consumed fuel cost
- Repair cost index
- Start-up cost
- Penalty cost

These costs, along with the model output costs, are listed in Table (7). In this table, the total cost, fuel cost, start-up cost, repair cost index, and the penalty cost are respectively shown as SUMi, Fi,Si, Ri, and Pi..

The values in Table (7) are also graphically shown in chart 1.

Table 6. Model output in LINGO

Objective value	HTS (hour)	Speed(rpm)	Selectee	Station name
1241074	45,65	7400,7400	G2,G3	Ghalejigh
	47.3,27.3	6373,6427	G1,G3	Neka
	148	7400	G1	Noor
	101.6,121.6	7400,7400	G3,G4	Ramsar

Table 7. Model outputs plus the costs associated with the ten operator selections

| S_i	R_i	P_i	F_i	SUM_i	Operator
33910	7.357	853841.5	527695.7	1415455	O1
34550	7.207	906210.6	560386.3	1501154	O2
41795	10.386	711197	508733.6	1261736	O3
43240	7.721	774975	557908.4	1376131	O4
49230	7.495	1100351	743142.8	1892731	O5
36254	7.314	716074.9	489114.1	1241450	O6
38756	7.314	775107.7	548146.9	1362018	O7
60790	10.311	1179317	839612.7	2079730	O8
45540	7.515	966265	643675.6	1655488	O9
52590	10.320	1064085	714819.1	1831504	O10
40375.14	7.442	711453.4	489237.6	1241074	(model output)O0
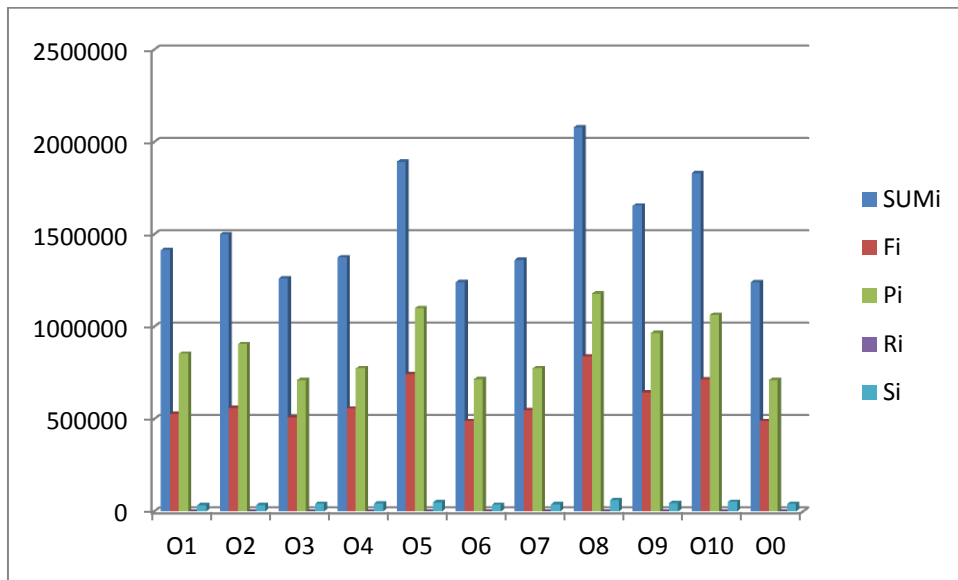
Chart .1. Model output values

The SUM$_i$ column in the Table (7) confirms that the model could minimize the total costs. The model optimizes total costs to the amount of 3%;however,some numerical values that are less than the model outputs draw attention. In other words, some options of the expert operators lower the costs in the F$_i$,S$_i$, R$_i$, and P$_i$ columns when compared to the model outputs. Therefore, the eight measureable scales are considered as selection strategies of options (model output and ten expert operators). They are the following:

1. Minimum total costs (the main objective function)
2. Minimum fuel cost
3. Minimum repair cost index
4. Minimum start-up cost
5. Minimum penalty cost
6. Minimum volumetric percentage of fuel cost
7. Minimum percentage of repair cost index plus penalty cost
8. Minimum percentage of start-up cost

If the significance weight of the eight measureable scales is assumed to be exactly the same, then the minimum value of each option dealing with every scale in the Table (7) would be found and would also be supposed to be the final choice of turbo compressors, which could be the model output or an operator's selections. However, referring to the upstream managers of the studied organization, including the chief executive officer, the overhaul repairing director, and the operation head officer, the eight scales are weighted in different values of significance. Thus, pursuing the AHP,the three experts were requested to show their ideas about the scales' importance in separate tables. Moreover, they were asked to fill in the pairwise matrix planned with the eight scales on each axis, to express the preference value of each scale with respect to the others. The relatively weighted values that were dedicated to the eight scales by the three experts are shown in Table (8). The inconsistency factor of each pairwise compared matrix filled in by the chief executive officer, the overhaul repairing director, and the operation head officer are respectively 0.09,0.08, and 0.09.

At this point, we have the expert's judgment and the relatively weighted value of each scale in hand. The geometric mean is also used to obtain the overall weighted value for each scale. The final weighted values of the scales and their ordering are shown in Table (9).

After the eight scale weighted values are determined, the numerical quantity of each option should be calculated for each scale. The values from Table (7) are used for this. Since the numbers of this table are relatively large, they are first normalized in each column using formula (43), to create smaller numbers.

$$R_N = \frac{(r - r_{min})}{(r_{max} - r_{min})} \qquad (43)$$

Table 8. Weights of scales (expert points of view)

Relative weight			Scale
Overhaul repairing director	Operation head	Chief executive	
0.252	0.293	0.400	Min. total costs
0.219	0.237	0.170	Min. fuel cost
0.243	0.127	0.178	Min. repair cost index
0.088	0.106	0.060	Min. start-up cost
0.031	0.039	0.027	Min. penalty cost
0.071	0.083	0.064	Min. volumetric percentage of fuel cost
0.049	0.061	0.052	Min. percentage of repair cost index plus penalty cost
0.047	0.054	0.048	Min. percentage of start-up cost

Table 9. Ranking of the overall weight of the eight scales

Ranking	Overall weight	Scale
1	0.315	Min. total costs
2	0.208	Min. fuel cost
3	0.181	Min. repair cost index
4	0.085	Min. start-up cost
8	0.032	Min. penalty cost
5	0.074	Min. volumetric percentage of fuel cost
6	0.054	Min. percentage of repair cost index plus penalty cost
7	0.051	Min. percentage of start-up cost

Finally, entering the normalized numbers in the "DATA GRID" section in "EC" software (and the relatively weighted value of each scale determined before) gives the absolute weight of each option. Because of the intrinsically minimizing total costs problem, all the options are from the smallest number ordered in Table (10).

At last, as shown in Table (10), there is an option for operators just like O6, who is the first candidate to select the turbo compressors to be turned on, despite the optimum minimization of total costs by the model. Consequently, the machines considered by operator O6 run to service to provide the customer demand.

Table. 10. Absolute weight of options and their ranking

Ranking	Absolute weight	Options
2	0.027	(O0) model output
3	0.042	O1
5	0.054	O2
7	0.082	O3
6	0.060	O4
9	0.145	O5
1	0.020	O6
4	0.045	O7
11	0.239	O8
8	0.101	O9
10	0.184	O10

7.Conclusion

Modeling of the operational costs of gas booster stations in a linear network by mixed integer nonlinear programming confirms the practicality and effectiveness of this mathematical technique. In the present research, the choice of turbo compressors, their speeds, their service times in terms of hours are considered as the decision variables, while the minimization of operational costs is the objective function. The operational constraints are mathematically stated. The outcomes of running the model in suitable optimization software were generated in an acceptable time and showed an improvement scaled at 3%relative to the operator's experimentally produced decisions. The nonlinear programming technique, in addition to suitably selecting active turbo compressors, would have favorable performance in terms of dedicating the appropriate rotary speed of the compressors in steady state operation, which is very significant for reducing the fuel cost of gas turbines as compressor drivers. The other important result obtained from the mathematical model output is the demonstration that the compressors running at higher speeds have a greater heating efficiency for the compressor's driver and a relatively greater polytrophic efficiency. Another consequence, when comparing the model output and the operator options, is the final selection of an operator's option that considers the important status of costs designated by managers and using the AHP process, despite the aim of total cost optimum minimization by the model. This issue could be seen as an important management issue that relies more on the experience and suggestions of skilled and educated staff. Furthermore, this is especially more noticeable in terms of the employees standing in the first queue of the operational zone of an organization, such as the gas booster station operators, and the chief officer can always have expert and trained operators as a decision-making team in the organization.

This research could be continued by complementary investigations, such as the following:

1. Deploying a tree or cyclic structure, rather than a linear one, for a gas distribution network.
2. The input/output pressures, input temperature, and gas flow rate transferring from each station in this study are deterministically assumed. The actual lack of crisp values for these parameters suggests the use of the fuzzy optimization method and comparison of the results.
3. Analysis of the model's sensitivity and checking the variability of the results beyond the changes in input parameters would be valuable investigations that would make this research more practical.

The compressibility factor Z is given the same value in the upstream and downstream areas of the pipeline to calculate the gas station service time and gas velocity formulas. However, due to the variation of this factor in different sections of the network, it should be replaced with more appropriate relations obtained from the gas pipeline hydraulic and fluid mechanic literature to give results that are closer to real conditions.

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