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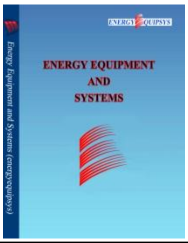

Economic optimization of solar systems in uncertain economic conditions using the Monte Carlo method

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ABSTRACT

Solar energy is an environmentally sustainable energy source as it is clean and inexhaustible. Solar systems are very common and cost-effective, thus, can be used for many home applications. In this paper, a new method is presented to optimize solar systems economically, regarding to energy cost fluctuations. In spite of conventional analyses, in which the inflation is considered constant, this method considers a probability distribution for inflation. The probability function of the life cycle solar saving (LCS) is then estimated by the Monte Carlo method. The expected value of LCS is used as the objective function. The standard domestic solar system is considered as a benchmark to show capability of the method. Three most important parameters of a solar water heating system are considered as manipulated variables. The optimal value of each parameter was found based on the proposed procedure, and employing the particle swarm optimization (PSO) algorithm as the optimization method. The results show that the collector area of 17 m^2, collector angle of 42^o, and storage tank of 100 l/m^2 maximize LCS to the mean value of 9930 USD for the selected case study. Also, the probability distribution of LCS shows that the mean value of the payback time is 4.1138 years with standard deviation of 1.3182.

Keywords: Economic Analysis, Monte Carlo, Optimization, Solar Energy.

1. Introduction

The widespread use of fossil fuels that are non-renewable leads to climate change, has adverse environmental impacts, and makes the exploration of sustainable and renewable energy sources more vital than ever.

In recent years, many studies have been done on the use of renewable energy technologies and their assessment. One of the factors that significantly affects the use of renewable energies is their uncertainty. This uncertainty can be the result of their changeable nature.

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For example, considering wind energy as a renewable energy, the wind speed varies due to weather conditions and many other parameters on a daily basis and during the year. The uncertainty can be attributed to the uncertainty in the cost assessment of the use of these resources. Some parameters such as amount of initial investment, saving of previous resources, and return of capital cost considered in economic analysis are strongly affected by uncertainty in economic parameters such as inflation rate or price of energy carriers

Renewable energy technologies usually come with higher power-specific upfront capital costs compared to the investments into conventional energy infrastructures [1, 2]. In general, capital costs are a function of the borrower's credit rating, the provided securities, the leverage ratio, and the aggregated project risk. Usually, higher aggregated project risk leads to higher interest rates demanded for the loan or even to the complete loan denial by lenders such as banks [3]. Furthermore, high transaction costs and other risks may prevent potential investors or institutional lenders to capitalize in renewable energy systems [1, 2]. Therefore, coping with financial constraints of renewable energy systems, investments require a stable and reliable political and legal framework so potential investors can reduce regulatory risks and hence significantly decrease the capital cost [4].

There are several methods to optimize the renewable energy systems. Most of the methods are based on maximizing economic benefits of these systems over their life time. To calculate economic benefits of these systems over a long period of time, it is essential to have an estimate for fuel price over these years. Most of the methods use a constant inflation for fuel price over these years. Looking at fuel price fluctuation over recent years shows that considering constant inflation is not a justified assumption (Fig. 1) [5]. There are a few works on economical consideration of energy and renewable energy systems in uncertainty of economic parameters. For example, de Silva Pereira et al proposed a method that utilizes the Monte Carlo method (MCM) to consider the uncertainty of variables' behavior using probability distribution functions [6]. They successfully applied this method on a Grid-Connected Photovoltaic System to analyze the risk of taking a specific decision [6]. Also, Arnold and Yildiz presented a new financial analysis. They demonstrated the new financial analysis combined with Monte Carlo Simulation aided in optimizing the conceptual design of an investment project with respect to capital returns and risk [3].Solar energy is one of the environmentally sustainable options that plays a significant role to replace non-renewable energy sources [7]. Solar water heating systems (SWHSs) are very common and cost-effective systems which can be used for many home applications [8]. SWHS contains conventional flat plate solar collectors (FPSCs) with a metal absorber plate and covers to convert incident solar radiation into heat and conduct it to a medium such as water [7].Several parameters, such as collector type, collector area, storage tank capacity, collector angle, fluid flow rate, and environmental conditions, affect the efficiency and cost of domestic solar systems. For example, oversizing the solar system increases the capital cost and rendering the system to be economically infeasible. On the other hand, under sizing the solar system reduces the solar fraction and increases the fuel cost of the auxiliary system [9]. Therefore, appropriate design of this system is necessary to guarantee an efficient operation and improve the economic feasibility of the solar system.

Several methods have been developed to design a SWHS. These methods can be generally classified as the correlation-based methods and simulation based methods. The utilizability method, f-chart method, and ϕf-chart method are the common correlation-

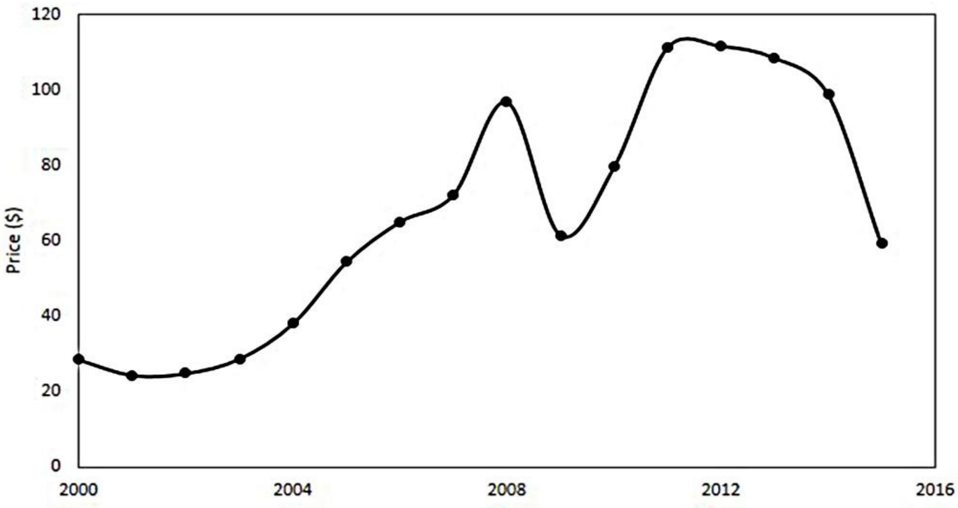

Fig. 1. Fuel (Brent crude oil) price Fluctuation [5]

based methods, in which the design parameters are usually given by the tables or charts provided by manufactures [8, 10]. F-chart method is the most common method which has been widely used in the literature [11]. Chang and Minardi compared the optimal collector area calculated by both TRNSYS program as simulation based method and f-chart method and found a good agreement between the results [12]. Also, Kalil Rahman et al. programmed the f-chart method and cost optimization techniques to determine the optimal number of collector panel for higher efficiency of the system [13]. Moreover, Kezhi et al calculated the optimal collector angle considering the monthly summation of auxiliary heat load with one year as the objective function. Their results exhibited that the optimal angle depends on the heat collector area or solar fraction [14].

Earlier, the optimal parameters were usually estimated by the operating performances such as the annual energy cost, internal rate of return, solar fraction, and design space. However, recently determining the optimal design considering life cycle analysis has increasingly attracted attention. In a life cycle analysis, besides the energy performance in the operation stage, the energy performances in all stages (e.g., production, operation, and maintenance) are taken into account [8]. Dennis Barely and Winn used f-chart program to approximate collector area which minimizes the total life cycle cost of an active domestic hot water heating system [15]. Also, Schroeder considered shadow effect of adjunct collectors on collector area and calculated the optimal collector area which maximizes life cycle saving [16]. Moreover, the collector area for SWHS was optimized based on life cycle cost analysis [9, 17]. In addition, Lima et al. estimated the optimal collector angle, collector area, and storage temperature based on life-cycle energy analysis using TRNSYS program as simulation based method [18].

In this study, a new method is proposed to optimize solar systems economically. A domestic SWHS in Los Angeles was considered as a benchmark to calculate optimal design of SWHS considering both life cycle analysis and risk analysis using Monte Carlo method. The annual thermal performance of solar active building heating system was estimated using f-chart method. The inflation rate was assumed to be inconstant and normally distributed over years. The probability distribution function was estimated by inflation data over the previous 45 years. The Monte Carlo method was then employed to assess probability function distribution of the life cycle solar saving. The expected value of the LCS was used as the objective function to find optimal values of the decision variables.

2.Problem definition

As mentioned, the objective of this study is to optimize the solar water heating system presented in Figure 2, economically. The SWHS contains solar thermal collectors, water tanks, interconnecting pipelines, and the water for transporting heat from the collector to storage. In this study, the collector area, storage capacity, collector type, collector angle, load, collector heat exchanger size, and fluid flow rate were considered as system parameters of design since they affect system efficiency.

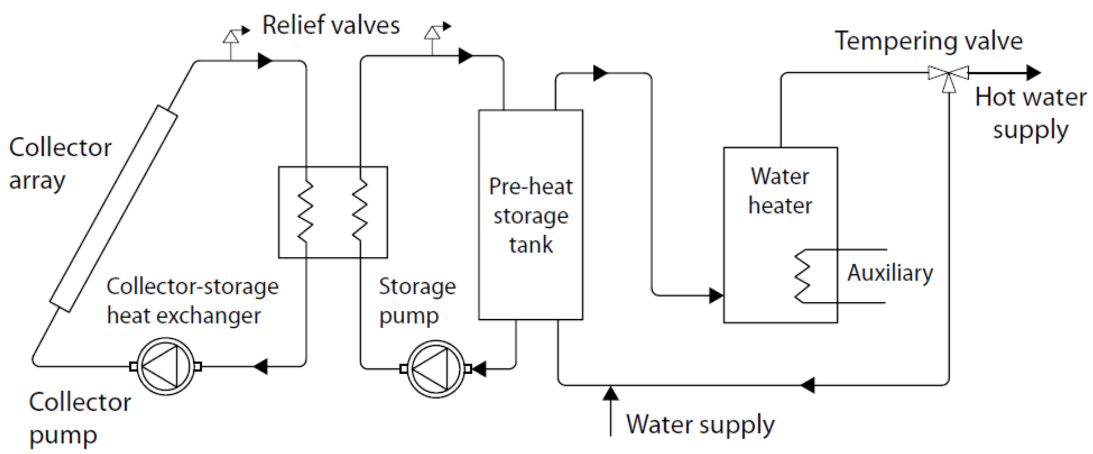

Fig. 2. Standard solar water heating system

3.Methodology

3.1. Economic Analysis

Generally, solar systems are associated by high initial cost and low operating costs since the energy source is free, but the equipment for collecting the solar energy and convert it to heat are costly. In order to make solar system economically feasible, it should be designed such that the costs of collectors, other required equipment, and conventional auxiliary fuel are lower than the cost of other conventional energy sources [19]. Therefore, an initial known investment along with present worth of estimated future operating costs, including both the cost to run and maintain the solar energy and auxiliary energy used as a backup should be considered. Other factors like the interest paid on money borrowed, inflation, taxes, insurance cost, and resale of equipment at the end of its life should also be considered. The objective of this economic analysis is to determine the optimal size of a solar system that gives the lowest combination of solar and auxiliary energy cost. The annual cost, S_a, was calculated by:

$$S_a = C_{mor} + C_{fuel} + C_{main} + C_{pe} + C_{pt} + C_{ins} - C_i \quad (1)$$

where C_{mor} is the annual mortgage payment, C_{fuel} is the annual fuel cost, C_{main} is the annual maintenance cost, C_{pe} is the annual

$$S_{ss} = C_{fs} - C_{emor} - C_{emain} - C_{epe} - C_{ept} - C_{eins} + C_i \quad (2)$$

where C_{fs} is the fuel saving, C_{emor} is the annual extra mortgage payment, C_{emain} is the annual extra maintenance cost, C_{epe} is the annual extra parasitic energy cost, C_{ept} is the annual extra property taxes, C_{eins} is the annual extra insurance cost, and C_i is the annual income tax saving.

These terms can be collected or subtracted if they are of the same type. In economics, costs are defined in three types; present time, future time, and periodic payments.

Generally, the present worth of an investment or cost at the end of n^{th} year, PW_n, at discount rate of d and constant inflation rate of i can be calculated by [19]:

$$PW_n = \frac{C(1+i)^{n-1}}{(1+d)^n} \quad (3)$$

Life cycle solar saving is defined as summation of present value of the solar saving and down payment.

Life cycle solar saving = present worth of solar saving-down payment (4)

3.2. Fuel Saving

To estimate the fuel saving, energy covered by solar energy, solar fraction has to be calculated. The f-chart method was used to calculate the annual thermal performance of solar active building heating system. The monthly fraction, f, of the total heating load, which can be supplied by solar energy system, was calculated using f-chart method [19]:

$$f = \frac{L_i - L_{AUX,i}}{L_i} = \frac{Q_{S,i}}{L_i} \quad (5)$$

where L_i is the monthly energy required by the load, $L_{AUX,i}$ is monthly energy required by the auxiliary, and $Q_{S,i}$ is the solar energy delivered.

In the analysis of standard solar water heating system, the fraction of the monthly load supplied by solar energy is a function of two dimensionless parameters. The first (X) is the ratio of collector losses to heating load which comprises both space heating and hot water load. The second (Y) is the ratio of absorbed solar radiation to the heating load [19].

$$f = 1.029Y - 0.065X_c - 0.245Y^2 + 0.0018{X_c}^2 + 0.0215Y^3 \quad (6)$$

where X and Y are:

$$X = F_R U_L \frac{\acute{F}_R}{F_R}(T_{ref} - \bar{T}_a)\Delta t \frac{A_C}{L} \quad (7)$$

$$X_C = X\left(\frac{11.6 * 1.18T_W + 3.86T_m - 2.32\bar{T}_a}{100 - \bar{T}_a}\right) \quad (8)$$

$$Y = F_R(\tau\alpha)_n \frac{\acute{F}_R}{F_R}\left|\frac{\overline{(\tau\alpha)}}{(\tau\alpha)_n}\right| \bar{H}_t N \frac{A_C}{L} \quad (9)$$

where X_c is the corrected value of X, F_R is the heat removal factor, $\acute{F}_R$ is the collector efficiency factor, U_L is the overall heat loss coefficient based on the collector area, T_{ref} is the reference temperature, Δt is time, A_C is the collector area, T_W is the minimum acceptable hot water temperature, T_m is the mains water temperature, $\overline{T}_a$ is the monthly average ambient temperature, L is the monthly heating load or demand, $(\tau\alpha)_n$ is the normal incident energy absorbed by collector, $\overline{(\tau\alpha)}$ is the monthly average value of absorbed over incident solar radiation, $\overline{H}_t$ is the monthly average daily total radiation on a tilted collector surface, and N is the number of days in a month. The fraction of annual load supplied by solar energy system, F, is calculated by [19]:

$$F = \frac{\sum f_i L_i}{\sum L_i} \tag{10}$$

where f_i is the monthly solar fraction.

To estimate such an optimal design, different collector area as a most effective parameter, storage volume and collector angle were manipulated and analyzed economically. It should be noted that all other parameters were kept constant.

3.3. Probability distribution functions

In most of the economic analyses, parameters such as interest rate, inflation, and cost of equipment used or energy produced are assumed to be constant, however, in long term economic analysis of a system, the uncertainty of variables and system should be considered. Therefore, a proper model should be developed to estimate inconstant parameters. When a forecasting model is developed (any model that plans ahead for the future) certain assumptions need to be made. These assumptions can be about the investment return on a portfolio, the cost of a construction project, or how long it will take to complete a certain task. Because these are projections into the future, the best which can be done is estimate the expected value.

In this study, inflation rate was considered inconstant. The inflation rate significantly affects investment return and subsequently affects the investor's decision making. The inflation rate of fuel cost in future was estimated based on probability distribution function estimated from historical data of fuel cost in the past 40 years.

3.4. Monte Carlo implementation

Monte Carlo method is a statistical sampling method that works with random components as input variables subjected to uncertainties, and after several iterations presents a set of results in terms of probabilities [6].

Selecting a probability distributions function and a good random number sequence generator are very important in the MCM. Also, the number of iterations plays an important role in the convergence of the method because as the number of iterations increases, the mean and the standard deviation of the samples tend to the average and the standard deviation of a normally distributed function [6].

The Monte Carlo method is used to estimate probability function of life cycle solar saving and payback time based on probability distribution function of the inflation. The expected value of the Monte Carlo results was then subjected to an optimization algorithm. The expected value of a discrete random variable with values of x_i and probabilities of p_i was calculated by the following equation:

$$E[X] = \sum_{i=1}^{\infty} x_i \, p_i \tag{11}$$

3.5. Optimization method

A wide variety of evolutionary algorithms (EAs) have been used to solve different types of optimization problems. Particle swarm optimization (PSO) algorithm, one of the major evolutionary global optimization algorithms, was used to minimize the objective function ($f(x)$) since it has shown a high convergence rate in multivariable problems [20, 21, and 22].

$$\min_{x} f(x) \qquad x \in S \subset \mathbb{R}^n \tag{12}$$

The PSO algorithm, first proposed by Kennedy and Eberhart (1995), starts with a randomly selected initial population and then successively evolves the individuals in a swarm:

$$x_i = x_{i-1} + v_i \tag{13}$$

$$v_i = c_0 v_{i-1} + c_1 r_1 \otimes (\hat{x}_i - x_{i-1}) + c_2 r_2 \otimes (\hat{g} - x_{i-1}) \tag{14}$$

where x is the position of an individual particle, v is the velocity that determines the displacement of the particle, i is the index for current iteration, c_0 is the inertia weight, c_1 and c_2 are the acceleration constants that control the influence of each of the velocity components, r_1 and r_2 are random vectors with the dimensionality of the search space, $\hat{x}$ is the particle's best-ever position, $\hat{g}$ is the swarm's best-ever position, and $\otimes$ stands for element-by-element vector multiplication. In other words, the position of an individual particle is updated by a displacement that depends on the particle's velocity of previous iteration, the best previous location of the particle (local best, *pbest*) and the best-ever location of the particle among all particles (global best, *gbest*) [19, 20].

However, the particles tend to move outside of the feasible boundary in the first few iteration [19]. Therefore, handling boundary constraints is required to achieve the best performance with PSO. Several methods for handling boundary constraints including random, reflecting, and absorbing methods have been proposed in the literature [19]. In this study, we have bounds on the values of the estimated parameters. In order to handle these bounds, when the particle flies outside of the boundary, the calculated displacement is divided by a factor, b, which is dynamically modified until the particle lies inside the boundary based on the following equations:

$$x_i = x_{i-1} + v_i/b_{ij} \qquad (15)$$
$$b_{ij} = \lambda\, b_{ij-1}$$

where λ is a constant value and is a problem dependent parameter. The algorithm flowchart of this method is exhibited in Figure 3.

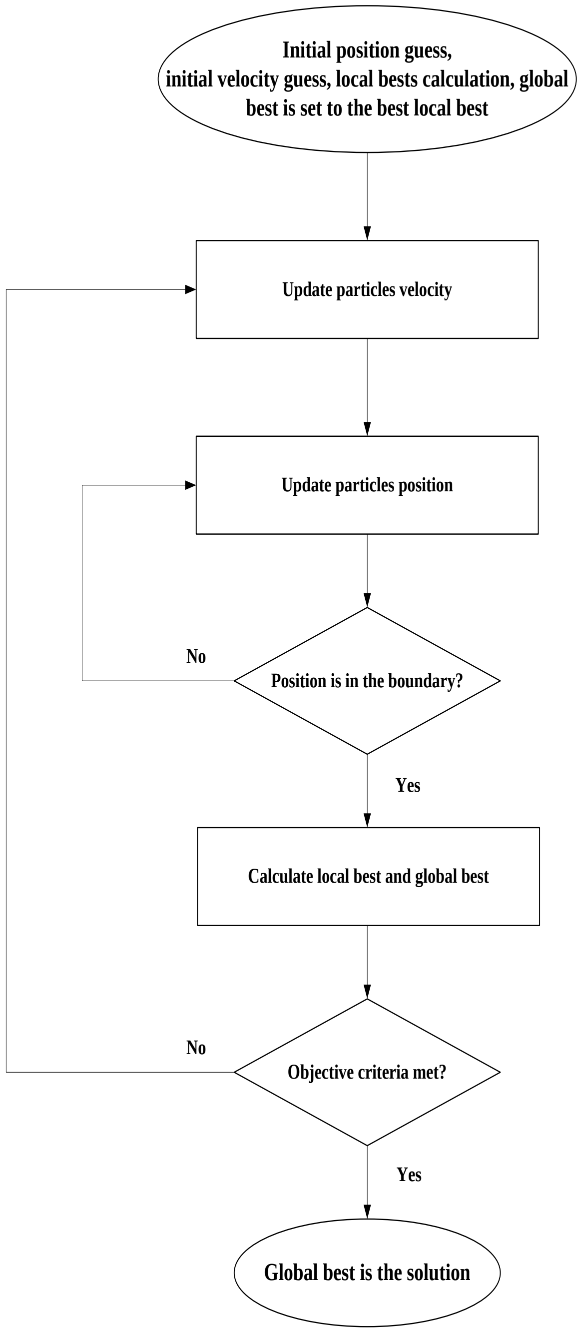

Fig. 3. PSO algorithm flowchart

3.6. Case study

A domestic solar water heating system in Los Angeles was considered as a case study to show the performance of the proposed method. Los Angeles (latitude 33.93°) was chosen because its monthly average radiation is good and, consequently, it is economically feasible.

This method was used to optimize the efficiency of the solar system as well as investment return. The characteristic parameters of the collector are shown in Table 1.

The life time of this system is 20 years and 70% of the initial cost is covered by a 10-year mortgage at an interest rate of 7% and at the end of the system life, the system will be sold for 30% of its original value. The general market discount rate is 8%. The maintenance, insurance, and parasitic energy costs, as well as property tax are not considered. Also, the fuel costs are expected to change at uncertain inflation rate per year. The results of this method are then compared with those of a usual method in which inflation is considered to be constant at 6.57% (average of past 45 years).

4.Results and Discussion

The normal distribution of inflation obtained by curve fitting of inflation in past 45 years is shown in Fig.4. It should be noted that the natural gas price from 1970 to 2014 (45 years) was used to calculate inflation rate in the past years [5].

The distribution function of life solar saving was then calculated. The PSO algorithm with 20 particles was used to calculate the optimal collector area, collector angle, and storage volume resulted in maximum expected value of life solar saving distribution function. The optimal parameters are presented in Table 2.

It should be noted that the parameters were subjected to lower and upper boundaries presented in Table 3. Also, optimal values of collector area, collector slope, and storage capacity were evaluated assuming constant inflation rate of 6.57% and presented in Table 2. As presented in this table, the optimal collector area and collector slope for inconstant inflation rate are higher than those for constant inflation rate. However, there is insignificant difference between the storage capacities of both scenarios.

To better understand the differences between the constant and inconstant inflation cases, the optimal system designs of both scenarios were compared. Two optimum systems obtained from the constant and

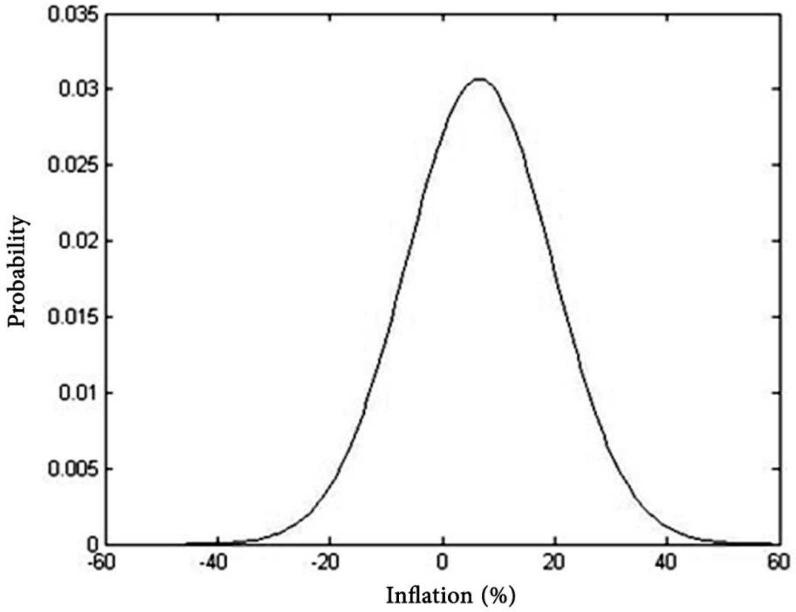

Fig. 4. Normal distribution of inflation generated by curve fitting

Table 1. Optimal designs for two scenarios

Inflation	collector area	collector slope	Storage capacity
Constant	12	40	100
Inconstant	17	42	101

Table 3. Lower and upper boundaries of manipulated parameters

Factors	Lower Boundary	Upper boundary	Unit
Collector area	10	30	m^2
Collector angle	30	60	°
Storage volume	100	150	l/m^2

inconstant cases were compared. The distribution of life solar saving for inconstant inflation optimal design is shown in Fig.5. The mean value of life solar saving is approximately 9930 USD. Also, the distribution of payback time for this scenario is presented in Fig.6. As shown in this figure, the average payback time is 4.1138 years.

In the next step, the probable inflations in future, predicted by previous inflation data, were used to analyze optimal system which was obtained from the constant inflation analysis.

The distribution of life cycle solar saving for constant inflation optimal design is shown in Fig.7. The mean value of life cycle solar saving is approximately 9170 USD. It can be concluded that inconstant inflation method increased life cycle solar saving by 7.6% compared to constant inflation case.

According to the objective function, maximizing life cycle solar saving, it was predictable that life cycle solar saving in the constant inflation case is less than the inconstant one.

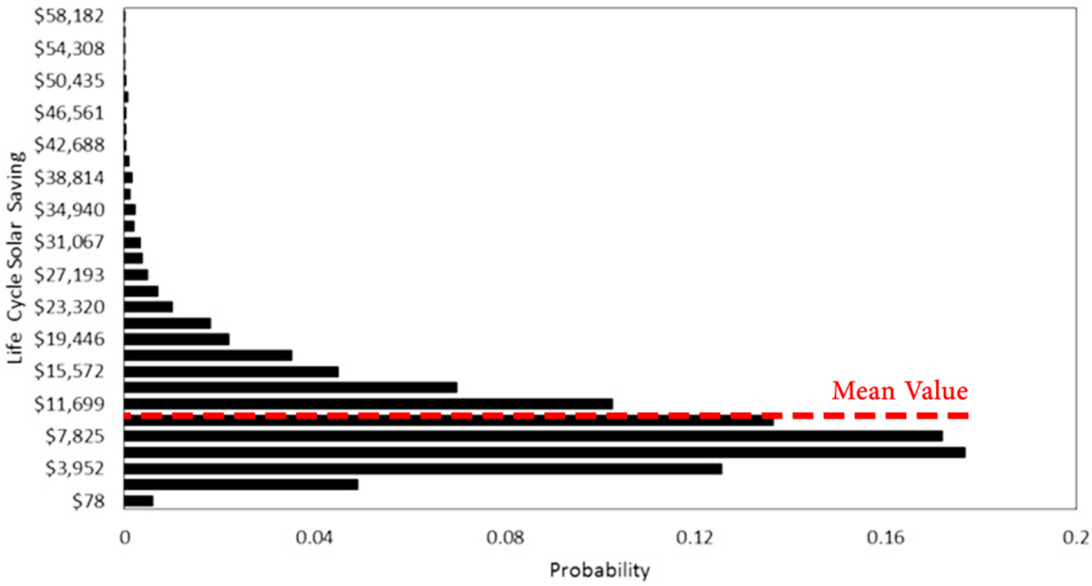

Fig. 5. Life cycle solar saving distribution for inconstant inflation- optimal design

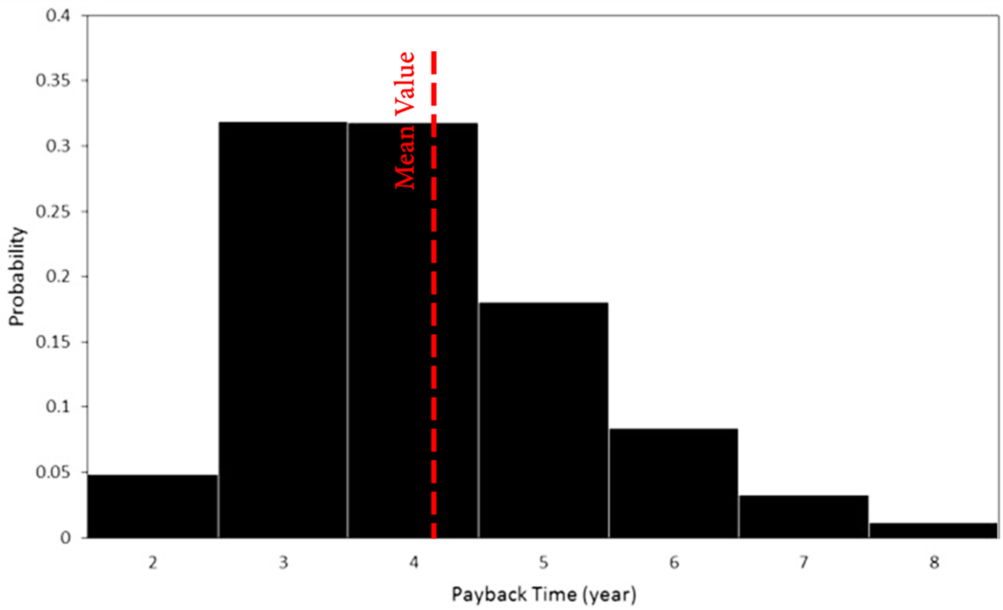

Fig. 6. Distribution function of payback time for inconstant inflation- optimal design

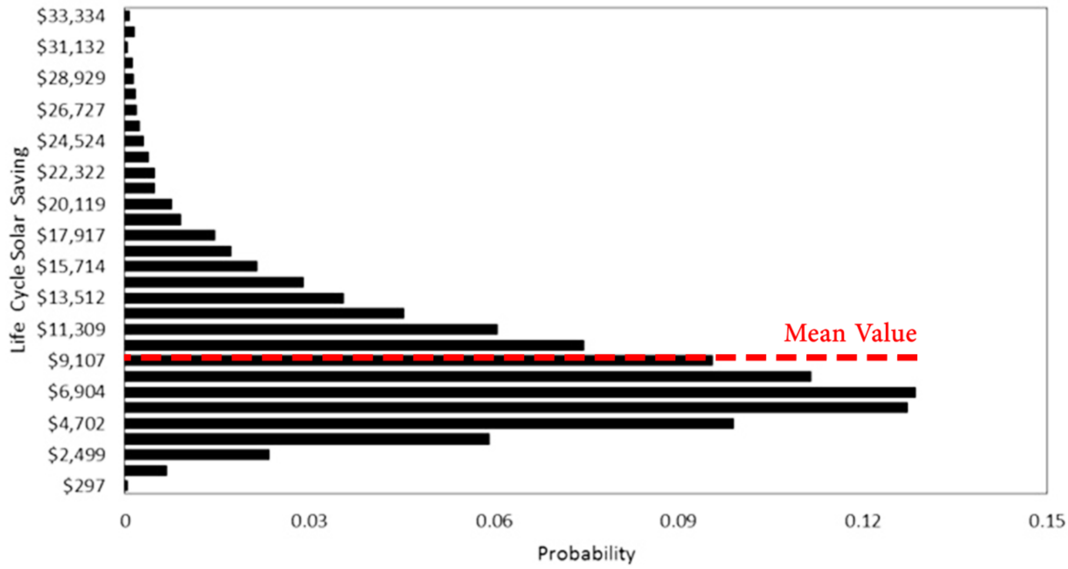

Fig. 7. Life cycle solar saving distribution for constant inflation- optimal design

The optimal design in the case of inconstant inflation is investigated in more detail since it showed better results compared to the optimal design of constant inflation case. As shown in Fig.8, the life cycle solar saving is in the range of 4900 to 14300 USD with 70% certainty and the probability of being less than 4900 or more than 14300 USD is going to be 15%. Also, as shown in Fig.9, payback time is in the range of 3 to 5 years with 80% certainty and the probability of being more than 5 years is 16%. These results can be used in decision making since they are most probable, more reliable, and safer results. The decisions according to these probable results can be much more useful and safer. The forecasting is going to be based on a range of possible values, instead of a single value and using a range of possible values gives a more realistic picture of what might happen in the future.

5. Conclusion

A new method for economic optimization of solar systems was proposed. In this study, life cycle analysis and risk analysis using Monte Carlo method were used to calculate optimal design of a bench mark solar water heating

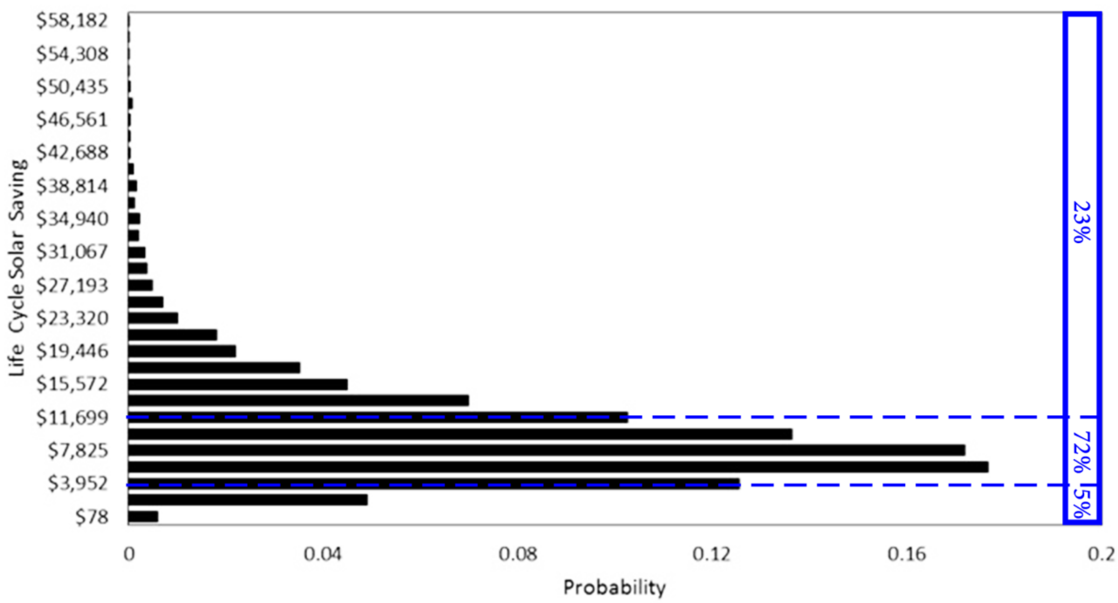

Fig. 8. Distribution function and integrated probability of life cycle solar saving for inconstant inflation- optimal design

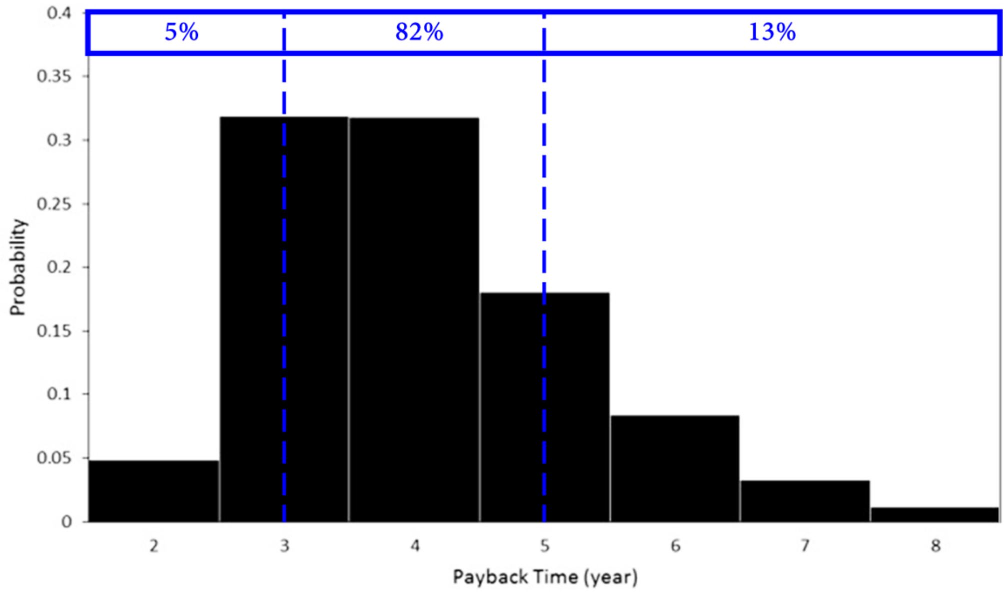

Fig. 9. Distribution function and integrated probability of payback time for inconstant inflation- optimal design

system. Despite the conventional economic methods, in which inflation was assumed constant, in this method, a probability distribution was considered for inflation. The probability distribution of life cycle solar saving and payback time were then estimated by the Monte Carlo method. The expected value of the life cycle solar saving was considered as the objective function and the optimal collector area, collector angle and storage volume were estimated. This optimal design was then compared to the optimal design of the conventional method considering constant inflation.

The results showed that considering inconstant inflation leads to 39% increase in solar collector's surface area while optimal values of the collector slope and storage capacity are not changed significantly. Also, the mean value of the life cycle solar saving in the optimal design of inconstant inflation case is improved by 7.6% compared to that of constant inflation rate. Moreover, the payback time is estimated between 3 and 5 years with 82% certainty, while life cycle solar saving is evaluated between 3900 to 11700 USD with 72% certainty which shows more uncertainty in estimation of life cycle solar saving compared to the payback time.

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